

10th IMACS International Symposium on Iterative Methods in Scientific Computing

Dedicated to Paul Van Dooren on his 60th Birthday

May 18-21, 2011

Marrakech , Morocco



Organized by

Université du Littoral Côte d'Opale, Calais , France

Université de Valenciennes et du Hainaut-Cambrésis , France

Université Cadi Ayyad, Marrakech, Morocco

Ecole Normale Supérieure, Rabat, Morocco

Kent State University, USA



Liberté • Égalité • Fraternité

RÉPUBLIQUE FRANÇAISE

AMBASSADE DE FRANCE AU MAROC



Fédération de Recherche
Mathématique
du Nord-Pas de Calais

TOPICS

The international conference "10th IMACS International Symposium on Iterative Methods in Scientific Computing " (IMACS 10) will be held in Marrakech, Morocco during May 18 - 21 , 2011. The meeting is dedicated to Professor Paul Van Dooren on his 60th birthday. The topics of this conference are (but not limited to):

- Large Linear Systems of Equations and Preconditioning,
- Large Scale Eigenvalue Problems,
- Saddle Points Problems,
- Domain Decomposition,
- Numerical Methods for PDEs,
- Optimization,
- High-Performance and Parallel Computation,
- Linear Algebra and Control,
- Model Reduction,
- Multigrid and Multilevel Methods,
- Applications to Image Processing, Financial Computation, Energy Minimization and Internet Search Engines...

INVITED SPEAKERS

- M. GANDER, University of Geneva, (Switzerland)
- V. MEHRMANN, Technische University Berlin, (Germany)
- G. MEURANT, (France)
- Y. SAAD, University of Minnesota, (USA)
- P. VAN DOOREN, Catholic University of Louvain, (Belgium)

SCIENTIFIC COMMITTEE

- M. BELLALIJ, - M. GANDER, - V. MEHRMANN, - M. PREVOST, - H. SADOK,
- A. H. BENTBIB, - K. JBILOU, - A. MESSAOUDI, - L. REICHEL, - P. VAN DOOREN,
- A. BOUHAMIDI, - A. LEMBARKI, - G. MEURANT, - Y. SAAD,

ORGANIZING COMMITTEE

- M. BELLALIJ , Université de Valenciennes, France
- A. H. BENTBIB, Faculté des Sciences et Techniques de Marrakech , Maroc
- A. BOUHAMIDI, Université du Littoral Côte d'Opale, France
- M. HEYOUNI, ENSA Al-Hoceima, Université Mohammed I, Oujda, Maroc
- K. JBILOU, Université du Littoral Côte d'Opale, France
- A. LEMBARKI, Université Cadi Ayad , Faculté des Sciences Semlalia, Marrakech , Maroc
- A. MESSAOUDI, ENS Rabat , Maroc
- M. PREVOST, Université du Littoral Côte d'Opale , France
- L. REICHEL, Kent State University , USA
- R. SADAQA, ENS Rabat, Maroc
- H. SADOK, Université du Littoral Côte d'Opale , France

List of plenary talks

Why it is so difficult to solve Helmholtz problems with classical iterative methods Martin J. Gander	3
Numerical solution of linear quadratic optimal control problems with differential-algebraic constraints Volker Mehrmann	4
Necessary and sufficient conditions for GMRES complete and partial stagnation G�rard Meurant	5
Multilevel methods for Ill-posed problems Lothar Reichel	6
Computing the diagonal of the inverse of a matrix Yousef Saad	7
Updating and downsizing the approximate eigenspace of an indefinite matrix Paul Van Dooren	8

List of abstracts

Alternative implementations of the hybrid Bi-CG methods for linear equations Kuniyoshi Abe	11
A new method for classification of the Brachiopods based on the Radon transformation Youssef Ait Khouya	12
Detection of slums in the region of Marrakech using genetic algorithms Mohammed Ait Oussous	13
Improving Newton's method for nonlinear optimization problems in several variables Ameen Alawneh	14
Mesh-free global and local radial basis collocation methods for three-dimensional partial differential equations Yasser Alhuri	15
A Game-theoretic approach to the minimization of the total transmit power for multiuser MIMO-OFDM systems Habib Ayad	16
The structured LU-Like decomposition and some of its applications Mustapha Bassour	17
Computing the GCD of two polynomials via Hankel and Bezout matrices Skander Belhaj	18
Polynomial algorithm for the maximum stable set problem in a new class of graphs Zineb Benmezziane	19
Blood flow modeling in the presence of a stent Youssef Bentaleb	20
A Novel model of credit scoring using hidden markov model Badreddine Benyacoub	21
Community detection for hierarchical video segmentation. Arnaud Browet	22
Localized potenPVD's contributions to numerical methods in systems and control Younès Chahlaoui	23
Gramian-based MoR of switched dynamical systems Younès Chahlaoui	24
A new cache efficient scheme for sparse matrix-dense matrix multiplication Eric Cox	25
A generalization of the Faber-Manteuffel theorem to rank-structured matrices Gianna M. Del Corso	26
Constrained non-negative matrix factorization. Application to industrial sources identification. Gilles Delmaire	27
Multigrid methods for zero-sum two player stochastic games with mean reward Sylvie Detournay	28

Construction, investigation and numerical resolution of high order accurate semi-discrete decomposition scheme for multidimensional quasi-linear evolution problem Nana Dikhaminjia	29
Sequences of Generalized Eigenproblems in DFT Edoardo Di Napoli	30
GA and ACO for the graph coloring problem Douiri Sidi Mohamed	31
Theoretical and numerical analysis of quasi-linear parabolic and periodic problem with strong nonlinearity with respect to the gradient Abdelwahab Elaassri	32
Boundary observability for semilinear hyperbolic systems Fatima Zahrae El Alaoui	33
Multiscales methods for transport phenomena Mohamed El Fatini	34
Triangular finite element simulation of nonlinear kinematic and diffusive wave problems Karima El Wassifi	35
Nonlinear iterative methods for regularization of inverse scattering problems in Banach spaces Claudio Estatico	36
Remarks on the preconditioned conjugate gradient method in the ocean general circulation model OPA Raffaele Farina	37
Perron vector optimization applied to search engines Olivier Fercoq	38
Solving eigenvalue problems with the dynamical functional particle method Marten Gulliksson	39
A meshless approximation method for solving the viscous Burger equation M. Hached	40
Local control of the quasi-interpolation error Maria José Ibanez Pérez	41
Parallelization of variable preconditioned Krylov subspace method with mixed precision using GPU Soichiro Ikuno	42
High-order discontinuous Galerkin method for two-layer shallow water equations: Application to the strait of Gibraltar Nouh Izem	43
Optimality conditions and duality in nondifferentiable multiobjective programming Izhar Ahmad	44
Backward error in linear least squares problems: estimates, their accuracy, and implementation Pavel Jiranek	45
Approach HUM for regional controllability of semilinear parabolic systems : state and gradient Asmae Kamal	46

A structured Symplectic SVD-Like decomposition Ahmed Kanber	47
Approximation of curves and/or surfaces by ODE variational splines Abdelouahed Kouibia	48
Purely algebraic domain decomposition methods for the incompressible Navier-Stokes equations Pawan Kumar	49
Adaptation of the M. N. El Tarazi theorem and application to the variable parallel overlapping domain decomposition method Mohamed Laaraj	50
On an algebraic optimized Schwarz preconditioning: performance and applications Lahcen Laayouni	51
Nonmonotone spectral projected gradient methods applying to Thomson problem Halima Lakhabab	52
Mathematical analysis of a system modeling ions electromigration through biological membranes Hamid Lefraich	53
Time domain decomposition with time reversible integration for initial value problem Patrice Linel	54
A dual decomposition method for color image restoration Cédric Loosli	55
On semifields of order q^4 : a computational approach Mashhour Al-Ali Bani Ata	56
A fast algorithm for solving Toeplitz plus Hankel structured total least squares problems Nicola Mastronardi	57
Stopping rules for iterative methods in nonnegatively constrained deconvolution Ornella Menchi	58
Inner-iteration GMRES methods for underdetermined least squares problems Keiichi Morikuni	59
Identification of Priestley-Taylor transpiration Parameters used in TSEB model by Genetic algorithm method over olive irrigated area. Abelhaq Mouida	60
A enhanced combined genetic algorithm-fuzzy logic controller (GA-FLC): application to free boundary problem Mourad Nachaoui	61
Preconditioning of systems arising from finite element discretizations of phase-field models Maya Neytcheva	62
Generalized circulant preconditioners for Toeplitz systems Silvia Noschese	63
A nonlinear four-point subdivision scheme Otheman Nouisser	64

Mathematical techniques for image retoration Fatima Zohra Nouri	65
Nearest stable polynomial using successive convex approximations Francois-Xavier Orban de Xivry	66
A fully-implicit Galerkin-characteristic method for nonlinear convection-dominated problems Ahmed-Naji Ould Esouid	67
An output controllability of bilinear systems: A numerical approach and simulations Maawiya Ould Sidi	68
Solvents of non-monic matrix polynomials Edgar Pereira	69
A domain decomposition method for the Canadian global numerical weather prediction model Abdessamad Qaddouri	70
A theory for preconditioning the inner-iteration in inexact inverse subspace iteration for generalized eigenproblems Mickael Robbé	71
Multiresolution analysis and supercompact multiwavelets for surfaces over non-uniform meshes Miguel L. Rodriguez	72
Existence of solutions for a seawater intrusion problem in a free aquifer Carole Rosier	73
Orthogonalization with a non-standard inner product with the application to approximate inverse preconditioning Miro Rozloznik	74
Quadratic Choreographies Philippe Ryckelynck	75
Töoeplitz minimal eigenvalues in signal and image processing Khalid Saeed	76
Error analysis of SRDECO Algorithm Ahmed Salam	77
Identification of parameters in hydrogeology: using genetic algorithm. Wenddabo Olivier Sawadogo	78
Discussion on optimal control via direct search optimization Emile Simon	79
Iterative refinement for symmetric saddle point problem Alicja Smoktunowicz	80
Parallel solution of problem with unilateral constraints Pierre Spiteri	81
Sequential and parallel methods for 4D anisotropic diffusion of dynamic PET images Clovis Tauber	82
A New iterative technique for solving nonlinear problems Helmi Temimi	83

Using proper orthonogonal decomposition in decouping dynamical systems Pham Toan	84
A MATLAB program of spectral dichotomy of regular matrix pencils Ahmed Touhami	85
On convergence of semi-discrete scheme for one nonlinear abstract hyperbolic equation Mikheil Tsiklauri	86
Evolutionary computation for optimal knots allocation in smoothing splines Olga Valenzuela	87
A local restart procedure for iterative projection methods for nonlinear large-scale eigenproblems Heinrich Voss	88
Implementation of scatter search to the scheduling problems Hanna Zini	89

Plenary Talks

Why it is so difficult to solve Helmholtz problems with classical iterative methods

Martin J. Gander *

Abstract

In contrast to the positive definite Helmholtz equation, the deceptively similar looking indefinite Helmholtz equation is difficult to solve using classical iterative methods. Applying directly a Krylov method to the discretized equations without preconditioning leads in general to stagnation and very large iteration counts. Using classical incomplete LU preconditioners can even make the situation worse. Classical domain decomposition and multigrid methods also fail to converge when applied to such systems.

The purpose of this presentation is to investigate in each case where the problems lie, and to explain why classical iterative methods have such difficulties to solve indefinite Helmholtz problems. I will also present remedies that have been proposed over the last decade, for incomplete LU type preconditioners, domain decomposition and also multigrid methods.

*Section de Mathématiques, Université de Genève, CP 64, Genève Switzerland

Numerical solution of linear quadratic optimal control problems with differential-algebraic constraints

Volker Mehrmann *

Abstract

We discuss the numerical solution of linear quadratic optimal control problems with constraints given by differential algebraic equations (DAEs). We show that the optimality system is a boundary value problem that is again a DAE and show how this can be solved to obtain optimal feedback solutions. This generalizes 30 year old results from Paul Van Dooren from the linear constant coefficient case to the linear variable coefficient and nonlinear case. We also discuss the numerical solution of the optimality boundary value problem.

*Institut für Mathematics, TU Berlin, Str. des 17. Juni 136, D610623, Berlin, Germany

Necessary and sufficient conditions for GMRES complete and partial stagnation

G erard Meurant *

Abstract

In this talk we will give necessary and sufficient conditions for the complete or partial stagnation of the GMRES iterative method for solving real linear systems. Our results rely on a paper by Arioli, Pt ak and Strako s (*Krylov sequences of maximal length and convergence of GMRES*, BIT Numerical Mathematics, v 38 n 4 (1998)), characterizing the matrices having a prescribed convergence curve for the residual norms. We will show that we have complete stagnation if and only if the matrix A is orthonormally similar to an upper or lower Hessenberg matrix having a particular first row or column or a particular last row or column. Partial stagnation is characterized by a particular pattern of the matrix Q in the QR factorization of the upper Hessenberg matrix generated by the Arnoldi process.

Since these conditions are not easy to check in practice, it is interesting to look for other characterizations. We will graphically illustrate the problem of complete stagnation for real matrices of order $n \leq 4$. For these matrices one can give necessary and sufficient conditions for the existence of a real right-hand side b such that the residual norms stagnate. We will also provide a sufficient condition for the non-existence of complete stagnation for a matrix A of any order n .

*gerard.meurant@gmail.com

Multilevel methods for Ill-posed problems

Lothar Reichel *

Abstract

Multilevel methods for the solution of well-posed problems, such as certain boundary value problems for partial differential equations and Fredholm integral equations of the second kind, are popular and their properties are well understood. Much less is known about the behavior of multilevel methods for the solution of linear ill-posed problems, such as Fredholm integral equations of the first kind with a right-hand side that is contaminated by error. We discuss properties of cascadic multilevel methods for the latter kind of problems.

*Department of Mathematical Sciences Kent State University Kent, OH 44242, USA

Computing the diagonal of the inverse of a matrix

Yousef Saad *

Abstract

Several emerging applications require to compute the diagonal of the inverse of a (sparse) matrix. These include Density Functional Theory in electronic structure calculations, Dynamic Mean Field Theory (DMFT), and uncertainty quantification. To solve this problem, a method based on probing can be used. This technique leads to the solution of many linear systems with different right-hand sides. Probing is quite effective when the diagonals of the inverse decay when moving away from the main diagonal, as happens when A is diagonally dominant. In other situations, alternative approaches must be considered. Among these we discuss techniques based on domain decomposition ideas.

*Minnesota Supercomputing Institute and Department of Computer Science and Engineering,
University of Minnesota, Minneapolis, MN 55455, USA

Updating and downsizing the approximate eigenspace of an indefinite matrix

Paul Van Dooren *

Abstract

Indefinite symmetric matrices occur in many applications, such as optimization, partial differential equations and variational problems where they are for instance linked to a so-called saddle point problem. In these applications one is typically interested in computing an estimate of the dominant eigenspace of such matrices, in order to solve regularized least squares problems or compute preconditioners. Let us consider a symmetric but indefinite matrix S_N . We limit ourselves to techniques where the approximations are computed incrementally for principal submatrices of S_N of growing dimension. Let S_N be an $N \times N$ symmetric matrix with eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_P > 0 > \lambda_{P+1} \geq \dots \geq \lambda_{N-1} \geq \lambda_N,$$

i.e., S_N has P positive eigenvalues and $N - P$ negative ones. Let S_n for $n < N$ be an $n \times n$ principal submatrix, i.e. $S_n := S(1:n, 1:n)$, $n = 1, \dots, N$, then its eigenvalues will interlace those of S_N and hence this matrix will also very likely be indefinite. We now want to compute rank m approximations A_n to these submatrices (with of course $m < n$) but also in factored form

$$S_n \approx A_n = U_n M_n U_n^T,$$

where $U_n^T U_n = I_n$, and $M_n \in \mathbb{R}^{m \times m}$ is a matrix of full rank m and where the m eigenvalues of M_n are good approximations of the m dominant ones of S_n (with that, we mean those of largest absolute value). In that case, U_n can also be expected to be a good approximation of the dominant eigenspace of S_n , provided its dominant eigenvalues are well separated from the rest of the spectrum.

The proposed method is an incremental method in which we compute an estimate of the dominant eigenbasis of matrices with growing dimensions. The method extends the results of [1] to the indefinite matrix case and is well-suited for large scale problems since it is efficient in terms of complexity as well as data management.

REFERENCES

- [1] N. Mastronardi, E. Tyrtshnikov, P. Van Dooren, *A fast algorithm for updating and tracking the dominant kernel principal components*, Siam J. Matr. Anal. Appl. 31 (2010) 2376-2399.

*CESAME, UCLouvain, Belgium. Joint work with Nicola Mastronardi, CNR, Sede di Bari, Italy.

Abstracts

Alternative implementations of the hybrid Bi-CG methods for linear equations

Kuniyoshi Abe *

Abstract

The hybrid Bi-Conjugate Gradient (Bi-CG) methods such as the Conjugate Gradient Squared method (CGS), the Bi-Conjugate Gradient STABILized method (BiCGSTAB), the BiCGstab(l) method and the Generalized Product-type method derived from Bi-CG (GPBiCG) are efficient solvers for solving a large sparse linear system. The Induced Dimension Reduction (s) method (IDR(s)) [3] has recently been developed, and moreover the equivalence between BiCGSTAB and IDR(s) for $s = 1$ is shown in [1]. Sleijpen et al. have reformulated BiCGSTAB to clarify the relationship between BiCGSTAB and IDR(s) in [1, 2]. The formulation of Bi-CG part used in the reformulated BiCGSTAB is different from that of the original Bi-CG; the coefficients α_k and β_k are computed by using a formulation that is closer to the IDR approach.

In this paper, we will redesign variants of CGS, BiCGSTAB, GPBiCG and BiCGstab(l) by using the same approach to compute Bi-CG part as was described in [1, 2], i.e., the formulation of Bi-CG described in [1, 2]. Here our redesigned BiCGSTAB and BiCGstab(l) will coincide with BiCGSTAB in [1] and IDRstab [2] for $s = 1$, respectively. Although these variants are mathematically equivalent to their counterparts, the computation of one of the Bi-CG coefficients differs, and the recurrences of our variants are also partly different from those of the original hybrid Bi-CG methods. This modification seems to allow a more accurate computation of the Bi-CG coefficients. Numerical experiments show that our variants are more stable and lead to faster convergence typically for linear systems for which the methods converge slowly (long stagnation phase).

REFERENCES

- [1] Sleijpen, G. L. G. Sonneveld, P. and van Gijzen, M. B., *Bi-CGSTAB as Induced Dimension Reduction Method*, Applied Numerical Math., 60 (2010) 1100-1114.
- [2] Sleijpen, G. L. G. and van Gijzen, *Exploiting BiCGstab(l) Strategies to Induce Dimension Reduction*, SIAM J. Sci. Comput., 32 (2010) 2687-2709.
- [3] Sonneveld, P. and van Gijzen, M. B., *IDR(s): A Family of Simple and Fast Algorithms for Solving Large Nonsymmetric Linear Systems*, SIAM J. Sci. Comput., 31 (2008) 1035-1062.

*Gifu Shotoku University, Gifu 500-8288, Japan. Joint work with Gerard L. G. Sleijpen, Utrecht university, Netherlands

A new method for classification of the Brachiopods based on the Radon transformation

Youssef Ait Khouya *

Abstract

The analysis is the quantification of the organization's morphology which forms an important aspect in the paleontological studies. On the one hand, they make it possible to understand the biodiversity in its morphological dimension. On the other hand, they highlight the morphological transformations undergone during the biological evolution. Historically, the form was encircled by a purely descriptive approach based on the qualitative evaluations of the morphological change starting from simple images. This approach was replaced gradually by the biometric methods having leads to the populated design of the fossil species. The variables used in such methods are linear dimensions, angles, surfaces and ratio or combination of dimensions. But, these biometric descriptors are insufficiently informative since they give only one approximate quantitative representation of the form and its changes. Then we used the Fourier analysis which consists in approximating the shape by a goniometrical function defined by a sum of terms of sine and cosine. This function is broken up into a series of amplitude of harmonics and phases or into a series of coefficient of Fourier being useful like variables for the quantitative analyses. But this method is valid right for the forms regular, indeed when morphologies become complex, it is not more possible to use such descriptors. In this paper, we propose a new method to identify Brachiopods based on the Radon transform. We use an adaptation of Radon transform called R-transform. Furthermore, to improve the uniqueness of the approach, a binary image is projected into the Radon space for different levels of the Chamfer distance transform. The advantages of the proposed method are robustness to noise, size and shift invariance.

*Joint work with M. Ait oussous and N. Alaa, Department of Mathematics, Laboratory of Applied Mathematics and Computer Science (LAMAI), Faculty of Science and Technology, BP 549, Marrakech, Morocco. e-mail: cimoinfo@hotmail.fr, alaanoureddine@gmail.com

Detection of slums in the region of Marrakech using genetic algorithms

Mohammed Ait Oussous *

Abstract

Active contours are powerful tools for image segmentation. Since the original work on snakes were introduced by Kass. Our approach has two stages: The first, using k nearest neighbour method for classification of the satellite image. The second, using the method of active contours by genetic algorithms formulation to detect the edge of the village. These algorithms can provide an interesting alternative when traditional optimization methods (method "Greedy", the variational approach, ...) do not provide reliable results

*Joint work with Y.Ait khouya, N.Alaa, Department of Mathematics, Laboratory of Mathematics and Computer Science (LAMAI), Faculty of Science and Technology, BP 549, Marrakech, Morocco
e-mail: youssef2005133@yahoo.fr, alaanouredine@gmail.com .

Improving Newton's method for nonlinear optimization problems in several variables

Ameen Alawneh *

Abstract

One of the most important problems in mathematics is solving systems of nonlinear equations. The problem of solving systems of nonlinear equations can be seen in finding the solutions of optimization problems. It is not an easy task to locate all critical points of a real valued function $f(X)$ on R^n by attempting to find exact solutions of $\nabla f(x)=0$. Instead iterative methods are used to search out the extremum by means of an approximating sequence whose points are generated in some computationally acceptable way from $f(x)$. More efficient methods for solving systems of nonlinear equations are continuously being sought. Some of these methods depend on variations of Newton's approach and spectral methods. Adomain decomposition method (ADM) was first introduced by Adomain and it's used for solving a wide range of problems. An advantage of ADM is provide analytical approximation or approximated solution to a wide class of nonlinear equations and stochastic equations without linearization, perturbation closer approximation or discretization. In this paper, ADM is used to improve Newton's method for minimizing function of several variables. Numerical solutions are calculated in the form of convergent series with easily computable components. The significant of this work is that improvement of Newton's method reduces computations, improves the accuracy and yields fast convergence.

*Dept of Math, Stat and Physics Qatar University, Doha - Qatar Email: aalawneh@qu.edu.qa

Mesh-free global and local radial basis collocation methods for three-dimensional partial differential equations

Yasser Alhuri *

Abstract

In the effort to develop schemes that treat arbitrary datasets, Mesh-free radial basis function (RBF) approximation has emerged as a popular choice, because it does not suffer from the need to impose its own geometrical structure (such as a grid or triangulation) in order to create an approximate. In short, RBF approximation is feasible regardless of the configuration of the dataset or domain of the target function. This technique has been successful in numerous and disparate real world problems involving the treatment of scattered data. The geosciences, computer graphics, and medical imaging are a few fields where RBFs have prospered. It is widely used in a variety of disciplines involving the processing of multivariate data, such as denoising, machine learning and the numerical solution of partial differential equations (PDEs). This study deals with the numerical solution of three-dimensional partial differential equations by the Mesh-free global and local radial basis collocation methods.

*yasseralhuri@yahoo.com -Hassan II University, FST, Mohammedia, Morocco

A Game-theoretic approach to the minimization of the total transmit power for multiuser MIMO-OFDM systems

Habib Ayad *

Abstract

A game theoretic approach to the minimization of the total transmit power for Multi-Input Multi-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) is proposed. the problem is cast as a strategic cooperative game with users as players, the set of allocated subcarriers as strategies and the total transmit power as payoff functions. The Nash equilibrium solution is established. In this work we assume that each user (player) has two strategic power allocations. those two strategic power allocation are based on two different power allocation algorithms. In our approach we assume that the channel state information (CSI) is perfectly known to the receiver and the transmitter, the singular value decomposition (SVD) is used to decompose the MIMO channel into a parallel eigenmode subchannels. In the simulation we present the characteristic and the performance of our approach, we investigate the impact of antennas number and users number and we compare our approach with the others algorithms.

*joint Work with Khalid El Baamrani: Equipe Tlcommunication et Rseaux informatiques, FSSM, Cadi Ayyad University, Marrakech, Morocco and
Abdellah Ait Ouahman: Laboratoire de Technologie de l'Information et Modelisation, ENSA of Marrakech, Cadi Ayyad University, Marrakech, Morocco
Corresponding author: ayad.habib@gmail.com

The structured LU -Like decomposition and some of its applications

Mustapha Bassour *

Abstract

We present in this work an algorithm that give a J -decomposition LU of an real $2n$ -by- $2n$ matrix M . The LU J -decomposition method is given in parallel with Gaussian elimination. The J -decomposition LU allows us to compute the factorization $R^J R$ of skew-Hamiltonian matrices. Decomposition $M = R^J R$ is the fundamental step for solving structured k^{th} degree polynomial eigenvalue problem $P(\lambda)v = \sum_{i=0}^k \lambda^i M_i v = 0$ which arise in many applications in science and engineering.

REFERENCES

- [1] NICHOLAS J. HIGHAM, *Solving Polynomial Eigenproblems by Linearization*, School of Mathematics, The University of Manchester Sackville Street, Manchester, M60 1QD, UK, March 3, 2005.
- [2] V. MEHRMANN AND D. WATKINS, *Polynomial eigenvalue problems with Hamiltonian structure*, Electronic Transactions on Numerical Analysis, Vol. 13, pp. 106-118, 2002.
- [3] HEIKE FASSBENDER, D. STEVEN MACKAY, NILOUFER MACKAY AND HONG-GUO XU, *Hamiltonian Square Roots of Skew-hamiltonian Matrices*. December 16, 1997.
- [4] P. BENNER, R. BYERS, H. FASSBENDER, V. MEHRMANN AND D. WATKINS, *Cholesky-like factorizations of skew-symmetric matrices*. Electr. Trans. Num. Anal., 11:85-93, 2000.

*Joint work with A. H. Bentbib, Faculté des Sciences et Techniques-Guéliz. Laboratoire de Mathématiques Appliquées et Informatique. BP 549 Marrakech, Morocco.(mustapha.bassour@gmail.com) / (bentbib@fstg-marrakech.ac.ma, ahbentbib@gmail.com)).

Computing the GCD of two polynomials via Hankel and Bezout matrices

Skander Belhaj *

The standard way to find the greatest common divisor (GCD) of two polynomials is via the Euclidean algorithm. Unfortunately, the classical algorithm turns out to be unstable when applied to approximate polynomials. One might think of designing stabilized numerical versions to the approximate case based on procedures involving matrices. This paper is principally concerned with the extension of the approximate approach for the block diagonalization of Hankel matrices [1] to the case of Bezout matrices. In fact, for the computation of polynomial GCD, the use of Bezout matrices seems better suited for numerical computations. We provide a comparison of the two approaches and show the better performances of the algorithms based on Bezoutians by means of an experimental evidence.

REFERENCES

- [1] S. Belhaj, *Computing the block factorization of complex Hankel matrices*, Computing 87 (3-4), (2010) 169-186.

*Université de Tunis El Manar, Ecole Nationale d'Ingénieurs de Tunis, Laboratoire LAMSIN, BP 37, 1002 Tunis, Tunisie. Email: skander.belhaj@lamsin.rnu.tn

Polynomial algorithm for the maximum stable set problem in a new class of graphs

Zineb Benmezziane *

Abstract

Finding the maximum independent set in a graph G is, in general, a difficult problem. And it remains NP-complete even for triangle-free or banner-free graphs. To solve this problem, various approaches were observed. And several authors have given iterative procedures, which at each stage, construct from a graph G another graph G' such that $\alpha(G') = \alpha(G) - k$, where k is a positive known integer.

Via pseudo boolean functions, Ebenegger and al. proposed a method with $k = 1$. This method was named Struction (Stability number RedUCTION).

The struction is a general approach to compute the stability number $\alpha(G)$ of a graph G . It construct iteratively a sequence of graphs $G_1, G_2, ..$ such that $\alpha(G_{i+1}) = \alpha(G_i) - 1$. But the main problem of this method is that the number of vertices of the graph G_{i+1} can increase exponentially, that is why the algorithm is not polynomial.

In this article, we give a modified algorithm of the struction and describe a new class of graphs Γ , for which the stability number can be obtained in polynomial time using this algorithm.

A graph in the class Γ is chair-free, net-free graphs and has the property that the subgraph induced by the set of centers of claws is the union of complete subgraphs.

Keywords: stability number, algorithm, polynomial, graph.

*Universit des Sciences et de la Technologie Houari Boumedienne, Alger 16111, Algrie; E-mail adress: zbenmezziane@yahoo.fr. Joint work with Moncef ABBAS

Blood flow modeling in the presence of a stent

Youssef Bentaleb *

Abstract

In this paper, we present a new approach based on the Navier-Stokes equations for modeling the blood-flow compartment in the artery with presence of the cardiovascular stent. In fact, we propose a mathematical one-dimensional (1d) model obtained making simplifying assumptions on solutions and to define the profiles of velocity and pressure liquid (blood) through the variable geometry of the arteries interest of simulation in this context is to make a comparison between different geometries without varying other parameters (such as blood flow, the properties of elasticity of the aortic wall). Our approach for solving optimization problems in complex geometry like the arteries, is to use first of methods and evolutionary algorithms, one-dimensional, based on a model that leads to equations of Navier-Stokes equations for estimating the velocity profile and blood pressure taking into account variations in the geometry pressure. We made numerical simulations with Comsol.

REFERENCES

- [1] Ataullakhanov, F. I. et Panteleev, M. A, Mathematical modeling and computer simulation in blood coagulation. Pathophysiology of Haemostasis and Thrombosis,(2005) 34(2-3) :60-70,
- [2] Petrilă, T. and Trif, D., Basics of fluid mechanics and introduction to computational fluid dynamics. Springer,(2005)
- [3] Sequeira, A. et Bodnar, T., Numerical simulation of the coagulation dynamics of blood. Computational and Mathematical Methods in Medicine, (2008) 9(2) :83-104,
- [4] Boivin. S, Cayr.F, . Hrad. J.M, A finite volume method to solve the NAVIER-STOKES equations for incompressible flows on unstructured meshes, Rapport EDF (1999) HE-41/99/002/k
- [5] Temam. R, Some developments on Navier-Stokes equations in the second half of the 20th century, Development of Mathematics 1950-2000, J-P. Pier Edition 2000.
- [6] Fuseri. B et Zagzoule CM-V , Etude physique et numérique de la circulation cérébrale en pathologie carotidienne CRAS Srie II, p 1039- 1047 1999

*joint work with Said El Hajji and Ghizlane Orhanou, Laboratory of Mathematics, Computing and Applications, Faculty of sciences Rabat, Morocco.

A Novel model of credit scoring using hidden markov model

Badreddine Benyacoub *

Abstract

Credit scoring is the set of decision models and their underlying techniques that aid lenders in the granting of consumer credit, Credit scoring is one the most successful applications of statistical modeling in finance and banking.

Hidden Markov models (HMM) are stochastic models capable of statistical learning and classification. They have been applied in many areas like speech recognition and handwriting recognition. In this study, HMM is used to evaluate and estimate the risk due to failure of a bank borrower. The result of evaluation of the risk assigns default bankruptcy probabilities to credit customers classifying them as good (solvent) and bad (insolvent) borrowers.

This paper investigates the estimation techniques of Hidden Markov Model to build a model for credit scoring in terms of classification and probability of default modeling. we propose a new algorithm of risk computation, where to make HMM a powerfel method for learning and decision as well as conventional methods like linear discriminant analysis and logistic regression.

REFERENCES

- [1] Robert J. Elliot, Alexei Filinkov : *A self tuning model for risk estimation*, Expert Systems with Applications 34(2008) 1692-1697.
- [2] Hassan Tahsin Oguz, Fikret S. Gurgun : *Credit Risk Analysis Using Hidden Markov Model*, IEEE Conferences: Computer and Information Sciences, 2008. ISCIS '08. 23rd International Symposium on (2008) Digital Object Identifier: 10.1109/ISCIS.2008.4717932.
- [3] Baesens,B : *Developing intelligent systems for credit scoring using machine learning techniques*, Ph.D Thesis, Katholieke Universiteit leuven (2003).
- [4] Elliot, R.J., Aggoum, L., Moore, J. B. : *Hidden Markov Models: Estimation and control*, New York: Springer (1995).

*Joint work with, Souad Bernoussi, A.Zoglat; Laboratory of Mathematic Informatics and Applications University Mohammed V-Agdal, Faculty of Sciences Rabat ,BP 1014, Morocco

Community detection for hierarchical video segmentation.

Arnaud Browet *

Abstract

In every day life, image and video processing becomes more and more important for example in urban traffic, medical analysis, surveillance but also within social networks. That explains why there has been a lot of scientific research around this topic for the last few decades.

In this work, we propose a new graph-based technique to uncover object contours and extend it to video tracking. First a large graph is built based on the individual pixels of a set of video frames. The weight of each edge, where each pixel corresponds to a node, depends on the distance between pixels (defined by the Euclidean or Chebyshev distance but also the distance in a well chosen color space or the optical flow). From this graph, we will show that good segmentation results can be achieved, without a-priori knowledge about the observed scene, by identifying each object of the picture as a community extracted from the graph. Communities have been initially defined by Newman and Girvan [1] as groups of highly connected nodes with only a few light connections between distinct groups. The authors defined a cost function Q , termed *modularity*, to assess the quality of a clustering in the graph:

$$Q = \sum_{ij} (W_{ij} - N_{ij}) \delta(c_j, c_j)$$

where W_{ij} is the edge weight between nodes i and j , N_{ij} is a well chosen null model to evaluate the strength of an edge and δ is a Kronecker delta indicating if nodes i and j are in the same community.

Optimizing the modularity has been proved to be NP-hard but we show that good segmentation can be achieved, at a reduced numerical cost, using an local version of the so-called Louvain method, recently proposed in [2]. This greedy algorithm recursively aggregated nodes or groups of nodes until no gain of modularity can be found.

REFERENCES

- [1] Newman, M. E. and Girvan, M., *Finding and evaluating community structure in networks*, Physical review. E, Statistical, nonlinear, and soft matter physics 69 (2004).
- [2] Browet, A., Absil, P.-A. and Van Dooren, P., *Community Detection for Hierarchical Image Segmentation*, IWCIA 2011.

*Joint work with P. Van Dooren and P.-A. Absil, universit  catholique de Louvain.

PVD's contributions to numerical methods in systems and control

Younès Chahlaoui *

Abstract

Paul M. Van Dooren received the engineering degree in computer science and the doctoral degree in applied sciences, both from the Katholieke Universiteit Leuven, Belgium, in 1974 and 1979, respectively. He held research and teaching positions at the Katholieke Universiteit Leuven (1974-1979), the University of Southern California (1978-1979), Stanford University (1979-1980), the Australian National University (1984), Philips Research Laboratory Belgium (1980-1991), the University of Illinois at Urbana-Champaign (1991-1994), Florida State University (1998) and the Universite Catholique de Louvain (1980-1991, 1994-now) where he is currently a professor of Mathematical Engineering. Dr. Van Dooren received the IBM-Belgium Informatics Award in 1974, the Householder Award in 1981 and the SIAM Wilkinson Prize of Numerical Analysis and Scientific Computing in 1989. He is a Fellow of IEEE and of SIAM (Society of Industrial and Applied Mathematics). He received the Francqui Chair in Antwerp in 2010. He is an Associate Editor of several journals in numerical analysis and systems and control theory. His main interests lie in the areas of numerical linear algebra, systems and control theory, and in numerical methods for large graphs and networks. In this talk we will review some of his main contributions and we will show some rare moments captured on pictures by some of his students and friends.

REFERENCES

- [1] Y. Chahlaoui, *Model reduction of switched dynamical systems*, 4th Conference on Trends in Applied Mathematics in Tunisia, Algeria and Morocco, Morocco, 6-8 May 2009. www.eprints.ma.man.ac.uk/1258
- [2] Z. Sun and S.S. Ge, *Switched linear systems. Control and design*, Communications and Control Engineering. London: Springer., 2005.
- [3] P. Van Dooren, *Gramian based model reduction of large-scale dynamical systems*. in Numerical Analysis 1999, 231–247, 2000.

*Centre for Interdisciplinary Computational and Dynamical Analysis (CICADA) School of Mathematics, The University of Manchester, Manchester, M13 9PL, UK. This work was supported by Engineering and Physical Sciences Research Council grant EP/E050441/1.

Gramian-based MoR of switched dynamical systems

Younès Chahlaoui *

Abstract

We consider a switched linear dynamical system described by

$$\begin{aligned}\delta x(t) &= A_\sigma x(t) + B_\sigma u(t), & x(t_0) &= x_0, \\ y(t) &= C_\sigma x(t),\end{aligned}\tag{1}$$

where $x(t)$ is the state, $u(t)$ is the controlled input, $y(t)$ is the measured output, σ is the piecewise constant signal taking values from an index set $M = \{1, \dots, l\}$, and A_k , B_k and C_k , $k \in M$ are matrices of appropriate dimensions. The switched system is a multi-model which is a special case of hybrid systems [2].

This talk is about model reduction of switched systems which has received relatively little attention in the Numerical Linear Algebra community. We will present several new Gramian-based methods. These Gramians are matrix energy functions and they are, in theory, solutions of certain complicated Lyapunov equations [3]. Here we propose to solve a set of simpler Lyapunov equations and to use linear combinations of these solutions to obtain the Gramians. We propose also a balanced truncation-like method with these two Gramians [1]. We will also present another new algorithm based on Lyapunov stability analysis. We will show how to solve the underlying set of Linear Matrix Inequalities for two common solutions. These two solutions are used to come up with a balanced truncation-like method. With this approach we will preserve the stability for the reduced model. We will suppose implicitly that each subsystem is stable.

REFERENCES

- [1] Y. Chahlaoui, *Model reduction of switched dynamical systems*, 4th Conference on Trends in Applied Mathematics in Tunisia, Algeria and Morocco, Morocco, 6-8 May 2009. www.eprints.ma.man.ac.uk/1258
- [2] Z. Sun and S.S. Ge, *Switched linear systems. Control and design*, Communications and Control Engineering. London: Springer., 2005.
- [3] P. Van Dooren, *Gramian based model reduction of large-scale dynamical systems*. in Numerical Analysis 1999, 231–247, 2000.

*Centre for Interdisciplinary Computational and Dynamical Analysis (CICADA) School of Mathematics, The University of Manchester, Manchester, M13 9PL, UK. This work was supported by Engineering and Physical Sciences Research Council grant EP/E050441/1.

A new cache efficient scheme for sparse matrix-dense matrix multiplication

Eric Cox *

Abstract

Solution of large sparse linear systems is often the most time consuming part of many science and engineering applications. Computational fluid dynamics, circuit simulation, structural mechanics, and computational nanoelectronics are just a few examples of the application areas, which give rise to solving large sparse linear systems. Sparse matrixvector (or -dense matrix) multiplications, in turn, are a major contributors to the time consumed by preconditioned iterative solvers. It is well-known that, unlike dense matrix operations, sparse matrix operations suffer significantly from excessive indirect memory addresses especially in deep memory hierarchies. In this paper we present a novel sparse matrix-dense matrix multiplication scheme that is based on a fast reordering strategy of the sparse matrix under consideration so as to take advantage of concentrating most of the work in the form of banded matrix (dense within the band) dense matrix multiplication. The resulting scheme minimizes the disadvantages of indirect memory addresses, is much more scalable, and is more tolerant of deep memory hierarchies. We present detailed performance comparisons with other well known sparse matrix multiplication kernels, including the well-known OSKI kernel of the Lawrence Berkeley National Laboratory.

*Joint work with Murat Manguoglu, Middle East Technical University, Faisal Saied and Ahmed Sameh, Purdue University

A generalization of the Faber-Manteuffel theorem to rank-structured matrices

Gianna M. Del Corso *

Abstract

In 1984 Faber and Manteuffel answered an important question posed in 1980 by Gene Golub about necessary and sufficient conditions on a matrix A for the existence of a three-term conjugate gradient type method for solving a linear system with A . In particular they showed that an $(s+2)$ -term conjugate gradient method exists if and only if A is normal of degree s that is A^H is a polynomial in A of degree at most s . The Faber-Manteuffel theorem can be formulated independently of conjugate gradient context as a result on the existence of a short recurrence for generating an orthogonal basis for the Krylov subspaces of A , or in terms of matrices giving necessary and sufficient condition for the reducibility of A to a banded-upper Hessenberg matrix (see [1]).

Theorem 1 [1] *Let A be an $n \times n$ matrix with minimal polynomial of degree $d_{\min}(A)$, let s be a nonnegative integer such that $s+2 < d_{\min}(A)$. A is normal(s) if and only if it can be reduced to a $(s+2)$ -band Hessenberg matrix.*

We investigate to what extent the hypothesis of the theorem can be relaxed to capture matrices that can be reduced to Hessenberg matrices having low rank above the s -th superdiagonal. In particular we proved the following theorem.

Theorem 2 *Let A be an $n \times n$ matrix, and let C be an $n \times n$ rank- k matrix. If s is the smallest integer such that there exists a polynomial $p_s(t)$ of degree s such that $A^H = p_s(A) + C$, then, A is reducible to an upper Hessenberg matrix having rank- k above the s -th superdiagonal.*

In the conjugate gradient context, for $k = 1$, we can reformulate Theorem 2 as follows.

Theorem 3 *Let A be a nonsingular matrix with minimal polynomial degree $d_{\min}(A)$. Let s be a nonnegative integer, $s+2 < d_{\min}(A)$. If there exists a polynomial $p_s(t)$ of degree s and a rank-one matrix C such that $A^H = p_s(A) + C$, then A admits, for any initial vector \mathbf{r}_0 a corrected optimal recurrence of length at most $s+2$, while for any \mathbf{r}_0 of grade with respect to A at least $s+2$, it admits a corrected optimal recurrence of length $(s+2)$.*

We are currently investigating how the Faber-Manteuffel proof can be modified to prove the necessary conditions.

REFERENCES

- [1] J. Liesen and Z. Strakoš, *On optimal short recurrences for generating orthogonal Krylov subspaces bases*, SIAM Rev. 50 (2008) 485-503.

*Joint work with Roberto Bevilacqua, Dipartimento di Informatica, Università di Pisa, Italy.

Constrained non-negative matrix factorization. Application to industrial sources identification.

Gilles Delmaire*

Abstract

Our contribution is concerned with factorization of an observation matrix into two unknown matrices, the contribution matrix G and the profile matrix F enabling to identify many pollution sources :

$$X = GF + E$$

The search for G and F may be achieved through factorization technics PMF (Positive Matrix Factorization) and NMF (Non negative Matrix Factorization) who look alternatively for best updates on G and F . These methods are sensitive to noise and initialization and give results up to a scaling factor and a permutation one. Weighted NMF [1] [2] have been proposed to take into account different standard deviations of the data matrix. However, some profiles components are inconsistent with practical experience. To avoid this drawback, constrained convex optimization is used in order to freeze some profile and contribution components and to let free the other ones. The problem is then equivalent to a family of constrained quadratic sub-problems whose solution may be computed. Constraints are expressed in our case as linear equality constraints where some components of the profile matrix are set to zero or to a constant positive value. Similarly, linear equality constraints may be added to the contribution matrix. Some global expressions including multiplicative updates of the whole matrixes are derived enabling a low computational load. These technics are used to estimate source contributions of air suspended particles. The challenge is to overcome difficulties coming from similar industrial profiles. On one hand, constrained optimization enables to avoid unexpected components in the profile matrix and on the other hand to get some more precise information on unknown profiles. For the same reasons, it provides a more precise contribution of the different sources on the whole air quality.

REFERENCES

- [1] Blondel V., Ngoc-Diep Ho and Van Dooren P., *Algorithms for Weighted Non Negative Matrix Factorization.*, Image and Vision Computing, submitted (2010) xx-xx.
- [2] Delmaire G., Roussel G., Hleis D. and Ledoux F., *Une version pondérée de la Factorisation Matricielle Non négative pour l'identification de sources de particules atmosphériques.*, European Journal of Automation (2010) n4-5, 547-566.

*Joint work with A. Limem and G. Roussel: (Lab. Inf. Sig. Image ULCO Calais Cedex, France) and D. Courcot: Unité de Chimie Environnementale et Interactions sur le Vivant ULCO. Dunkerque, France.

Multigrid methods for zero-sum two player stochastic games with mean reward

Sylvie Detournay *

Abstract

We develop a fast numerical algorithm for large scale zero-sum two player stochastic games with perfect information and mean reward, which combines policy iteration and algebraic multigrid methods.

Consider a finite state space \mathcal{X} . The stochastic game is played in stages as follows. The initial state x_0 is given and known by the two players. The player who plays first, says MAX, chooses an action in a set of possible actions. Then the second player, called MIN, chooses an action in another set of possible actions. The actions of both players and the current state determine a payment made by MIN to MAX at stage 0 and the probability of the new state x_1 . Then the game continues in the same way with state x_1 and so on. We call a strategy or policy for a player, a rule which tells him the action to choose in any situation. A Markovian strategy depends only, possibly randomly, on the current state. Each pair of Markovian stationary strategies of the two players determines a Markov chain on \mathcal{X} . We are studying the value of the game with mean reward which is defined as the mean expected payment per stage, made by MIN to MAX, when each player chooses a strategy maximizing his reward.

The value of the game is solution of a dynamic programming equation. This nonlinear equation can be solved by the policy iteration algorithm for zero sum stochastic games of Hoffman and Karp (66) when the Markov transition matrices of the game are all irreducible. The principle of this algorithm consists in applying successively the two following steps: first compute the value of the game with fixed strategy for the first player and then improve this strategy. The first step is solved applying the policy iteration for one player games, i.e. stochastic control problems. Cochet-Terrasson and Gaubert (06) proposed a version of policy iteration for two player games in the general multichain case, which is based on the algorithm for multichain Markov decision processes of Denardo and Fox (68). Each iteration of Denardo and Fox algorithm requires the computation of stationary probabilities of irreducible Markov chains and also the solution of linear systems of the type $v = Mv + r$ where M is a sub-markovian matrix.

We propose an algorithm based on Cochet-Terrasson and Gaubert policy iteration algorithm where we use multigrid methods for Markov chains of Horton (94) and De Sterck and al (08) to find the stationary probabilities and algebraic multigrid algorithm of Ruge and Stüben (86) for the above linear systems. We present numerical results of this algorithm (implemented in C) for large scale zero-sum games.

*Joint work with Marianne Akian, Inria Saclay and CMAP Ecole Polytechnique, France.

Construction, investigation and numerical resolution of high order accurate semi-discrete decomposition scheme for multidimensional quasi-linear evolution problem

Nana Dikhaminjia *

Abstract

In the present work there is considered the following nonlinear evolution problem:

$$u'(t) + Au(t) + M(u(t)) = f(t), \quad t > 0, \quad u(0) = \varphi.$$

Here A is a self-adjoint positively defined operator in Hilbert space H and $A = A_1 + A_2 + \dots + A_m$ ($m \geq 2$), where A_1, A_2, \dots, A_m are self-adjoint positively defined operators. φ is a given vector from $D(A)$, $f(t)$ is a continuously differentiable function, nonlinear operator $M(\cdot)$ satisfies Lipschitz condition. Let us introduce the following net domain $\bar{\omega}_\tau = \{t_k = k\tau, k = 0, 1, \dots, \tau > 0\}$. On the basis of high order accurate rational splitting of the semigroup (see [1]), there is constructed the following fourth order of accurate decomposition scheme:

$$u_{k+1} = V(2\tau)u_{k-1} + \tau(g_{k+1} + 4V(\tau)g_k + V(2\tau)g_{k-1}),$$

where $g_k = (f(t_k) - M(u_k))/3$,

$$\begin{aligned} 2V(\tau) &= T_\tau(\alpha)\bar{T}_\tau(\bar{\alpha})T_\tau(\bar{\alpha})\bar{T}_\tau(\alpha) + \bar{T}_\tau(\alpha)T_\tau(\bar{\alpha})\bar{T}_\tau(\bar{\alpha})T_\tau(\alpha), \\ T_\tau(\alpha) &= W_\tau(\alpha A_1) \dots W_\tau(\alpha A_m), \quad \bar{T}_\tau(\alpha) = W_\tau(\alpha A_m) \dots W_\tau(\alpha A_1), \\ W_\tau(A) &= (I - \alpha\tau A)(I + \bar{\alpha}\tau A)^{-1}(I - \bar{\alpha}\tau A)(I + \alpha\tau A)^{-1}, \quad 4\alpha = 1 \pm i/\sqrt{3}. \end{aligned}$$

Numerical realization of the constructed scheme on each time layer t_{k+1} is carried out using the following iterative process:

$$\begin{aligned} u_{k+1}^{(m)} &= -\frac{\tau}{3}M(u_{k+1}^{(m-1)}) + F_k, \quad k = 1, 2, \dots, \quad m - \text{iteration index}, \\ F_k &= V(2\tau)(u_{k-1} + \tau g_{k-1}) + 4\tau V(\tau)g_k + \frac{\tau}{3}f(t_{k+1}). \end{aligned}$$

The stability of the decomposition scheme is investigated and the error of the approximate solution is estimated. Using this scheme, there are carried out numerical calculations for different model problems. On the basis of the results of numerical calculations there are studied the stability and accurate order of the obtained decomposition scheme.

REFERENCES

- [1] J. Rogava, M. Tsiklauri, *High Order Accuracy Decomposition Schemes for Evolution Problem*, Lecture Notes of TICMI (2006), vol. 7, p. 164.

*Ilia Vekua Institute of Applied Mathematics, 2 University Street, 0186, Tbilisi, Georgia .
Joint work with Jemal Rogava (Ilia Vekua Institute of Applied Mathematics, 2 University Street, 0186, Tbilisi, Georgia) and Mikheil Tsiklauri (Ilia State University, Kakutsa Cholokashvili Ave 3/5 Tbilisi 0162, Georgia)

Sequences of generalized eigenproblems in DFT

Edoardo Di Napoli*

Abstract

Research in several branches of chemistry and material science relies on large numerical simulations. Many of these simulations are based on Density Functional Theory (DFT) models that lead to sequences of generalized eigenproblems $\{P_i\}$. Every simulation normally requires the solution of hundreds of sequences, each comprising dozens of large and dense eigenproblems; in addition, the problems at iteration $i + 1$ of $\{P_i\}$ are constructed manipulating the solution of the problems at iteration i . The size of each problem ranges from 10 to 40 thousand and the interest lays in the eigenpairs corresponding to the lower 10-30% part of the spectrum. Due to the dense nature of the eigenproblems and the large portion of the spectrum requested, iterative solvers are not competitive; as a consequence, current simulation codes uniquely use direct methods.

In this talk we present a study that highlights how eigenproblems in successive iterations are strongly correlated to one another. In order to understand this result, we need to stress the importance of the basis wave functions, which constitute the building blocks of any DFT scheme. Indeed, the matrix entries of each problem in $\{P_i\}$ are calculated through superposition integrals of a set of basis wave functions. Moreover, the state wave functions—describing the quantum states of the material—are linear combinations of basis wave functions with coefficients given by the eigenvectors of the problem. Since a new set of basis wave functions is determined at each iteration of the simulation, the eigenvectors between adjacent iterations are only loosely linked with one another. In light of these considerations it is surprising to find such a deep correlation between the eigenvectors of successive problems.

We set up a mechanism to track the evolution over iterations $i = 1, \dots, n$ of the angle between eigenvectors $x^{(i)}$ and $x^{(i+1)}$ corresponding to the j^{th} eigenvalue. In all cases the angles decrease noticeably after the first few iterations and become almost negligible, even though the overall simulation is not close to convergence. Even the state of the art direct eigensolvers cannot exploit this behavior in the solutions.

In contrast, we propose a 2-step approach in which the use of direct methods is limited to the first few iterations, while iterative methods are employed for the rest of the sequence. The choice of the iterative solver is dictated by the large number of eigenpairs required in the simulation. For this reason we envision the Subspace Iterations Method—despite its slow convergence rate—to be the method of choice. Nested at the core of the method lays an inner loop $V \leftarrow AV$; due to the observed correlation between eigenvectors, the convergence is reached in a limited number of steps. In summary, we propose evidence in favor of a mixed solver in which direct and iterative methods are combined together.

*Joint work with Paolo Bientinesi, AICES, RWTH Aachen, Germany.

GA and ACO for the graph coloring problem

Douiri Sidi Mohamed *

Abstract

Let $G = (V, E)$ an undirected graph, V corresponds to the set of vertices and E corresponds to the set of edges. The graph coloring problem is to associate a color to each vertex so that two connected vertices do not have the same color. A valid k -coloring of vertices in a graph $G = (V, E)$ is an application $c : V \rightarrow \{1, \dots, k\}$ such as $c(x) \neq c(y), \forall (x, y) \in E$, the value $c(x)$ associated with vertex x is called color of x (Figure 1).

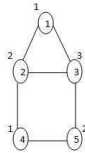


Figure 1: G coloring in 3 colors.

(GCP) allows to model some applications of the operational research such as the timetable problems, warehouse management, scheduling problem, etc. Several methods and algorithms are proposed to resolve the problem of coloring, there are exact methods and heuristics [1][2]. The smallest number of different colors used for a valid coloring is called the chromatic number, and denoted $\chi(G)$. It is well known that the k -coloring problem is NP-completeness and the $\chi(G)$ -coloring is NP-hard, thus heuristic approaches are inevitable in practice. In this paper we propose two metaheuristics for solving (GCP), the genetic algorithms (GA) and an algorithm of optimization by colonies of ants (ACO), to give approached values of $\chi(G)$ and compare the results found by the two metaheuristics.

REFERENCES

- [1] C. Lucet, F. Mendes, A. Moukrim, *An Exact Method for Graph Coloring*, Computers and Operations Research (2006) volume 33, number 8, pp. 2189-2207.
- [2] I. Blochliker, N. Zufferey, *A graph coloring heuristic using partial solutions and a reactive tabu scheme*, Computers and Operations Research (2008) vol. 35, no. 3, pp. 960-975.

*Le Laboratoire de Recherche Mathématiques, Informatique et Applications, Faculté des Sciences Rabat Maroc; douirisidimohamed@hotmail.fr.

Joint work with , Souad El Bernoussi, Faculté des Sciences Rabat Maroc; souad.elbernoussi@gmail.com

Theoretical and numerical analysis of quasi-linear parabolic and periodic problem with strong nonlinearity with respect to the gradient

Abdelwahab Elaassri *

Abstract

In this work we present two mathematical models of the quenching process. The first is based on the parabolic equation of heat conduction and has a variable temperature. In the second model, we reformulate the problem with the internal energy as a variable. We obtain a parabolic equation with strong nonlinearity in the gradient of this energy and periodic in time. To prove the existence of weak solution the latter model, we truncate the equation obtained. The classical techniques based on upper and lower solution can be applied and ensure the existence of periodic solution for the approached problem. The difficulty back to show that the solution of the approximate problem converges to the solution of our system. It is the primary objective of this work. The second objective in this work is the numerical simulation of this model. We use the multiple shooting method, and we reformulate the discrete problems in terms of optimization. Numerical examples are presented and commented.

References

- [1] N. Alaa, I. Mounir, Global existence for some quasilinear parabolic Reaction-Diffusion systems with mass control and critical growth with respect to the gradient, *Journal of Mathematical Analysis and Application* 253, 532-557, 2001.
- [2] M. Avrami, *Kinetics of phase change*, *Journal Chem. Phys* (1939) 1103-1112.
- [3] J. Deuel and P. Hess, Nonlinear Parabolic Boundary Value Problems with Upper and Lower Solutions, *Israel Journal of Mathematics*. Vol. 29, N°1, (1978).

*Joint work with N. Alaa, Faculté des Sciences et Techniques Gueliz, Laboratoire LAMAI . B.P.549 Marrakech-Maroc. elaassri_abdelwahab@yahoo.fr; alaa@fstg-marrakech.ac.ma

Boundary observability for semilinear hyperbolic systems

Fatima Zahrae El Alaoui *

Abstract

The study of non-linear systems which model the real problems is difficult compared to linear ones. The regional observability is a very important notion allows the study of such systems. The notion of linear systems and fixed point techniques are used in this work to solve the problem of regional observability for semi-linear hyperbolic systems . It consists to observe the boundary initial state of semi-linear hyperbolic systems in a part of the boundary of the system evolution domain. We give definitions and we use two reconstruction approaches based on extension of Hilbert Uniqueness Method and Sectorial property of the considered operator. The obtained results lead to an algorithm which is tested with numerical example and simulations.

REFERENCES

- [1] T. Kato, *Perturbation theory of linear operators*, Springer Verlag (1978).
- [2] E. Zerrik, S. Benhadid and H. Bourray. Sensors and boundary state reconstruction of hyperbolic systems. *Int. J. Appl. Math. Computer Sci.* vol. 20, no. 2, 227-238. (2010).
- [3] E. Zerrik, R. Larhissi and H. Bourray, *An output controllability problem for semi linear distributed hyperbolic system.* *Journal of Applied Mathematics and Computer Science* (2007). vol. 17, no 4.
- [4] E. Zerrik, S. Benhadid and H. Bourray, *Sensors and regional observability of the wave equation*, *Sensors and Actuators Journal* (2007), vol. 138, no. 2.

*Joint work with A. Boutoulout, H. Bourray, Department of Mathematics and Computer Sciences. Moulay Ismail University, Faculty of Sciences. Mekns. Morocco

Multiscales methods for transport phenomena

Mohamed El Fatini *

Abstract

We derive an error estimates for multiscales stabilized method of transport problems. The error estimator is efficient and reliable and depends only on the discrete solution, the mesh-size and the data of the problem.

More recently stabilized methods have been reformulated in the context of the variational multiscale formulation. Such decomposition allows splitting the weak form of the problem into two sub-problems: one for the coarse scales and one for the subscales. Examples of these methods are: Residual-Free Bubbles (RFB) [1], Multiscale Finite Element Method (MFEM) [4], Subgrid Stabilization [3] and Orthogonal subscale stabilization (OSS) [2]. Our purpose is to combine stabilization and a posteriori error estimators to increase the stability of the solution.

REFERENCES

- [1] F. Brezzi and A. Russo. Choosing bubbles for advection–diffusion problems. *Math. Models Meth. Appl. Sci.*, 1994; **4**: 571–587.
- [2] R. Codina, On stabilized finite element methods for linear systems of convection-diffusion-reaction equations. *Comp. Meth. Appl. Mech. Engrg.*, 2000; **188**: 61-82.
- [3] J.L. Guermond. Stabilization of Galerkin approximations of transport equations by subgrid modeling. *ESAIM, Math. Mod. Numer. Anal.*, 1999 **33**(6):1293–1316.
- [4] T. J. R. Hughes, Multiscale phenomena: Green’s functions, the Dirichlet-to-Neumann formulation, subgrid-scale models, bubbles and the origin of stabilized methods. *Comput. Methods Appl. Mech. Engrg.*, 1995; **127**:387–401.

*Université de Pau et des pays de l’Adour, IPRA, LMA, Avenue de l’université, BP 576, 64012 Pau, France. E-mail: melfatini@gmail.com

Triangular finite element simulation of nonlinear kinematic and diffusive wave problems

Karima El Wassifi *

Abstract

Generally the hydrodynamic models of overland flow are based on the shallow water wave theory described by the Saint-Venant (SV) equations. These models are derived from either the Kinematic wave (KW) or the diffusion wave (DW) approximations.

In our study, we developed two models to solve one and two dimensional Kinematic and Diffusion wave equations (Advection-Diffusion equations) which simulate overland runoff and find out runoff at the watershed, by a Galerkin triangular Finite Element Method (FEM). Particular emphasis is placed two numerical schemes which are first and second order accurate in space and time: Implicit and Crank-Nicholson schemes respectively. The nonlinear system equation is solved using Newton-Raphson iteration. The study presents also the difficulties and remedies of Galerkin finite elements method when there are discontinuities in the solution for the pure convection problems.

Test cases for both one-and two-dimensional problems, compare the numerical solution of the (DW) and (KW) models with analytical solutions for special cases. These simulations show that the proposed numerical models accurately predict the overland flow for several situations.

*Laboratoire de Mathématiques Appliquées et Informatique (LAMAI), Faculté des Sciences et Techniques, Université Cadi Ayyad, Marrakech; elwassifikarima@yahoo.fr
Joint work R. Khiri and N. Alaa, Université Cadi Ayyad; A. Ouahsine, UTC-LHN, Laboratoire Roberval, Compiègne Cedex, France and H. Smaoui, CETMEF-LHN, Laboratoire Roberval, Compiègne Cedex, France.

Nonlinear iterative methods for regularization of inverse scattering problems in Banach spaces

Claudio Estatico *

Abstract

Formally, any inverse problem can be modeled by an operator equation $Ax = y$, where $A : X \rightarrow Y$ is the so-called “forward” operator between two functional spaces X and Y , and $x \in X$ is the “cause” of some “effects” $y \in Y$. By means of the knowledge of (some approximation of) the effects y , the aim is to recover the unknown cause x . Many well known solving schemes approximate the solution x by means of the minimization of a cost functional like $\Phi(x) = \|Ax - y\|_Y + \lambda \|x\|_X$, where X and Y are both Hilbert spaces, such as the classical L^2 space, so that any $\|\bullet\|$ is basically the classical Euclidean norm. Iterative regularization algorithms for the minimization of Φ give rise in general to over-smoothed solutions and the discontinuities present in real solutions x are not well restored.

More recently, it has been investigated the behavior of iterative methods based on more general Banach spaces X and Y , such as, for instance L^p , with $1 < p < +\infty$ [1]. This way, the “size” of both the residual $Ax - y$ and the restored signal x are measured by means of the metric of the involved Banach spaces. The new “geometry” of the Banach spaces can substantially reduce the over-smoothing of the iterative restoration process. For instance, the norm of the space L^p mainly emphasizes, for values of the constant $1 < p < 2$, the “weight” of the small components, so that the minimization of Φ leads to a stronger reduction of such a small values.

The direct generalization to Banach spaces of any classical linear iterative algorithm for Hilbert spaces give rise to non-linear algorithms. In this talk, we discuss the behavior of a non-linear iterative method for regularization in Banach spaces. The algorithm is applied to a non-linear inverse scattering problem where the dielectric distributions x of a $2D$ domain must be recovered by means of its scattered microwave field y outside the domain. We will show how the new computational results well outperform classical “Hilbertian regularization”.

REFERENCES

- [1] F. Schoepfer, A.K. Louis, Th. Schuster *Nonlinear iterative methods for linear ill-posed problems in Banach spaces*, Inverse Problems, 22 (2006/01), 311-329.

*Department of Physics and Mathematics, University of Insubria, Como, Italy. E-mail claudio.estatico@uninsubria.it

Joint work with Matteo Pastorino and Andrea Randazzo, Department of Biophysical and Electronic Engineering, University of Genova, Italy. E-mail (matteo.pastorino@unige.it; andrea.randazzo@unige.it)

Remarks on the preconditioned conjugate gradient method in the ocean general circulation model OPA

Raffaele Farina *

Abstract

In this work we analyze the elliptic kernel's solver of the numerical model **OPA** that is a Ocean General Circulation Model. In OPA we have the following Laplace's problem:

$$\begin{cases} \frac{\partial}{\partial x} \left[\alpha(x, y) \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial t} \right) \right] + \frac{\partial}{\partial y} \left[\beta(x, y) \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial t} \right) \right] = f(x, y, t) & \text{su } \Omega \times [t_0, t_1] \\ \frac{\partial \psi}{\partial t} = 0 & \text{su } \delta\Omega \times [t_0, t_1] \end{cases} \quad (1)$$

The problem's (1) discretization gives n linear systems $E x = b_i$ $i = 1, \dots, n$ that are solved to determine the dynamic and the thermodynamic variables of ocean fluid at time t_1 . The aim of this work is to optimize the algorithm of Preconditioned Conjugate Gradient Method used by Ocean Numerical Model **OPA** to resolve the n linear systems. We replace the existent diagonal preconditioner P^{-1} with the preconditioner \bar{P}^{-1} obtained by the incomplete Cholesky's decomposition of E , in order to avoid the low convergence of the algorithm of Preconditioned Conjugate Gradient Method in **OPA**. This behavior depends on the domain grid resolution and on the relationship between the functions α and β . We theoretically and numerically show an increasing of the performance in terms of higher speed of convergence with a computational cost that scale linearly with the size of the problem.

REFERENCES

- [1] Madec G, Delecluse P, Imbard M, Levy C *OPA 8.1 general circulation model reference manual*, Notes de l'IPSL, Universit P. et M. Curie, B102 T15-E5, 4 place Jussieu, Paris cedex 5, N11, 91p
- [2] Saad Y., *Iterative Methods for Sparse Linear Systems 2nd*, Society for Industrial and Applied Mathematics Philadelphia, PA, USA 2003 ISBN:0898715342
- [3] Penduff, T., J. Le Sommer, B. Barnier, A.-M. Trguier, J.-M. Molines and G. Madec, *Influence of numerical schemes on current-topography interactions in 1/4 global ocean simulations*, 2007 , Ocean Science, 3, 509-524

*Department of Mathematic and Applications "R. Caccippoli" - University of Naples Federico II email:raffaele.farina@unina.it. Joint work with Salvatore Cuomo, Department of Mathematic and Applications "R. Caccippoli", University of Naples Federico II, email:salvatore.cuomo@unina.it .

Perron vector optimization applied to search engines

Olivier Fercoq *

Abstract

Internet search engines use a variety of algorithms to sort web pages based on the text content of the pages or on the hyperlink structure of the web. We consider here the case of link-based algorithms and we study the optimization of the ranking of a given web site. The problem consists in finding an optimal outlink strategy subject to design constraints and for a given search engine.

In a previous work with Akian, Bouhtou and Gaubert [2], we studied the PageRank optimization problem. In this case, the ranking is given by the invariant measure of a stochastic matrix. Here, we consider the more general situation in which the ranking is determined by the Perron eigenvector of a nonnegative, but not necessarily stochastic, matrix. Thus we cover Kleinberg's HITS algorithm. For both above problems, we have a concise description of the convex hull of admissible matrices. So we consider convex sets of matrices, but we show that Perron vector optimization on convex sets is NP-hard. However, we provide an efficient algorithm for the computation of the derivative of the criterion. This allows us to design a first order method giving a local minimum.

Unlike general Perron vector optimization problems, PageRank optimization problems have a Markov decision process structure. This yields a very efficient algorithm converging to a global optimum. We also identify assumptions under which there exists a "master" page to which all controlled pages should point.

Finally, we report numerical results on fragments of the real web graph for these search engine optimization problems.

REFERENCES

- [1] E. Deutsch and M. Neumann. *On the first and second order derivatives of the perron vector*. Linear algebra and its applications, 71:57–76, 1985.
- [2] O. Fercoq, M. Akian, M. Bouhtou, and S. Gaubert. *Ergodic control and polyhedral approaches to Pagerank optimization*. arXiv preprint:1011.2348, 2010.
- [3] C. de Kerchove, L. Ninove and P. van Dooren. *Maximizing PageRank via outlinks*. Linear Algebra and its Applications, 429(5-6):1254–1276, 2008.
- [4] J. Kleinberg. *Authoritative sources in a hyperlinked environment*. Journal of the ACM, 46:604–632, 1999.
- [5] T. Matsui. *NP-hardness of linear multiplicative programming and related problems*. Journal of Global Optimization, 9: 113–119, 1996.

*Centre de Mathématiques Appliquées Ecole Polytechnique 91128 Palaiseau Cedex France; INRIA, France; email: olivier.fercoq@inria.fr

Solving eigenvalue problems with the dynamical functional particle method

Marten Gulliksson *

Abstract

We consider the linear eigenvalue problem $A\mathbf{v} - \lambda\mathbf{v} = 0$ where $A \in \mathbb{R}^{n \times n}$, $\lambda \in \mathbb{C}$, $\mathbf{v} \in \mathbb{R}^n$, and $\|\mathbf{v}\|_2 = 1$ with the standard 2-norm in \mathbb{R}^n . The eigenvalue problem is solved by the Dynamical Particle Functional Method, DFPM, which consists of solving the dynamical system

$$M\ddot{\mathbf{u}} + N\dot{\mathbf{u}} = A\mathbf{u} - (\mathbf{u}^T A \mathbf{u}) \mathbf{u}, \|\mathbf{u}\|_2 = 1 \quad (1)$$

where $M = \text{diag}(\mu_1, \dots, \mu_n)$, $N = \text{diag}(\eta_1, \dots, \eta_n)$ and $\mu_i > 0, \eta_i > 0, i = 1, \dots, n$. We

show that the solution to (1) converges asymptotically to the eigenvector corresponding to the largest eigenvalue if there is at least one real eigenvalue. The smallest eigenvalue is attained by solving $-(A\mathbf{v} - \lambda\mathbf{v}) = 0$. Furthermore, we show that the convergence rate is exponential independent of problem size. With deflation, all eigenvalues may be attained. The time-independent Schrödinger equation was used to test the method where $A = A^T$ is sparse and the smallest eigenvalue is determined. For this problem we compare DFPM with the standard software packages ARPACK and LAPACK. These tests show that DFPM is approximately 10 times faster for a sparse matrix of size $n = 10000$ and 50 times faster for a sparse matrix of size $n = 500000$ respectively.

*Joint work with Sverker Edvardsson. Department of Natural Sciences, Mid Sweden University, Sweden

A meshless approximation method for solving the viscous Burgers equation

M. Hached *

Abstract

In this talk, we discuss meshless radial basis function methods for solving the Burgers equation with the Dirichlet boundary conditions given by

$$\begin{aligned}\frac{\partial u(x,t)}{\partial t} &= \nu \Delta u(x,t) - u(x,t) \cdot \nabla u(x,t) + f(x,t), \quad \text{for } x \in \Omega, t \geq 0, \\ u(x,t) &= g(x,t), \quad \text{for } x \in \partial\Omega, t \geq 0. \\ u(x,0) &= u_0(x), \quad x \in \partial\Omega\end{aligned}$$

An approximation of the solution in the domain is given as a radial basis function with time-dependent coefficients. Taking in account of the initial condition leads to a large-scale nonlinear ordinary differential equation (ODE). So, we use an implicit Runge-Kutta method with a high stage to solve a such ODE. The main drawback of such methods is the cost required at each integration step for computing the solution of a nonlinear system of equations. We will show how to reduce the cost of the computation by transforming the linear systems arising in the application of Newtons method to matrix equations. We propose an iterative projection method onto block Krylov subspaces for solving numerically such matrix equations. Numerical examples are given to illustrate the performance of our proposed method.

REFERENCES

- [1] M.D. Buhmann, *Radial Basis Functions*, Cambridge University Press, Cambridge, 2003.
- [2] A. Bouhamidi, *Weighted thin plate splines*, Anal. Appl. 3 (3) (2005) 297324.
- [3] A. Bouhamidi and K. Jbilou, *Stein implicit RungeKutta methods with high stage order for large-scale ordinary differential equations*, Applied Numerical Mathematics, 61(2011) 149-159.
- [4] A. Bouhamidi and K. Jbilou, *Meshless thin plate spline methods for the modified Helmholtz equation*, Comput. Methods Appl. Mech. Engrg. 197 (2008) 37333741.
- [5] A. El Guennouni, K. Jbilou, A.J. Riquet, *Block Krylov subspace methods for solving large Sylvester equations*, Numer. Alg. 29 (2002) 7596.

*Joint work with A. Bouhamidi and K. Jbilou,

LMPA, Université du Littoral Côte d'Opale, 50 Rue F. Buisson, BP 699, 62228 Calais- Cedex, FRANCE

Local control of the quasi-interpolation error

María José Ibáñez Pérez *

Abstract

The cubic quasi-interpolant [2, p. 104]

$$Q_h f = \sum_{i \in \mathbb{Z}} \frac{1}{6} (-f((i-1)h) + 8f(ih) - f((i+1)h)) M_4\left(\frac{\cdot}{h} - i\right)$$

provides a spline approximant for a function f defined on the real line from its values on the grid $h\mathbb{Z}$, $h > 0$. For a regular enough function f , the estimate $f - Q_h f = O(h^4)$ holds, and $Q_h f$ is a good approximation of f in practice.

Let us suppose that the goal $|f - Q_h f| \leq \varepsilon$ it is required on an interval I for a given tolerance $\varepsilon > 0$. If there exists a subinterval J of I such that $|f(x) - Q_h f(x)| > \varepsilon$ for $x \in J$ and $|f(x) - Q_h f(x)| \leq \varepsilon$ for $x \in I \setminus J$, the condition on the quasi-interpolation error is not satisfied, then we look for an approximant $A_h f$ such that (a) the operator A_h is exact on \mathbb{P}_3 ; (b) $A_h f = Q_h f$ on I ; (c) and $|f(x) - A_h f(x)| < \varepsilon$ when $x \in J$.

We will propose and analyze the construction of such operator A_h by using near-best operators (see e.g. [1]) or by regularizing the Heaviside's function.

REFERENCES

- [1] D. Barrera, M. J. Ibáñez, P. Sablonnière and D. Sbibih, *Near minimally normed spline quasi-interpolants on uniform partitions*, J. Comput. Appl. Math. 181 (2005) 211–233.
- [2] C. K. Chui, *An introduction to wavelets*, Academic Press, San Diego, 1992.

*Joint work with D. Barrera, Departement of Applied Mathematics, University of Granada, Faculty of Sciences, Campus de Fuentenueva s/n, 18071-Granada, Spain.

Parallelization of variable preconditioned Krylov subspace method with mixed precision using GPU

Soichiro Ikuno *

Abstract

The Variable Preconditioned Krylov subspace method with mixed precision on Graphics Processing Unit (GPU) using CUDA [1] is numerically investigated.

Recently, the performance of GPU surpasses that of CPU and various researches of General Purpose computing on GPU (GPGPU) have been proposed aggressively. However, GPU can calculate in the case of single precision fast, but becomes slow in double precision calculation because GPU is a device used for drawing of the graphics. Although the performance in double precision is still superior to the other commodity processors, high performance single precision operations of GPU should be used effectively.

K. Abe *et al.* developed new preconditioning strategy which is called the Variable Preconditioned Generalized Conjugate Residual (VPGCR) method [2]. In VPGCR, the residual equation is solved in each iteration instead of preconditioned matrix calculation. The convergence theorem of VPGCR is guaranteed that the residual of VPGCR converges if the relative residual norm of variable precondition calculation r satisfies the criterion $r < 1$ in each iteration. The residual equation can be solved in the range of single precision, which means that VPGCR is applicable method to elicit the high performance of GPU.

In this study, we apply the hybrid scheme that uses single precision and double precision operations to VPGCR method using GPU. Note that JOR method is adopted for precondition process. In the precondition process, single precision operations can be used for faster calculation since only approximate results are needed. The reduction of execution time in the precondition that occupies a large part of calculation is dramatically effective although the main process must be executed in double precision as same as usual.

Result of computation shows that VPGCR with mixed precision on GPU demonstrates significant achievement than that of normal VPGCR on CPU. VPGCR with mixed precision on GPU is 5.89 times faster than that of normal VPGCR on CPU.

REFERENCES

- [1] http://www.nvidia.com/object/cuda_develop.html
- [2] K. Abe and S. L. Zhang, *Int. J. Numer. Anal. Model.* **22** (2005) p. 147.

*Tokyo University of Technology, 1404-1 Katakura, Hachioji, Tokyo, Japan. Joint work with Y. Kawaguchi, Tokyo University of Technology, Tokyo, T. Itoh, Seikei University, Kichijoji-kitamachi, Musashino, Tokyo, Japan and K. Watanabe, Hokkaido University, Sapporo, Japan.

High-order discontinuous Galerkin method for two-layer shallow water equations: Application to the strait of Gibraltar

Nouh Izem*

Abstract

This study presents a discontinuous Galerkin finite-element method to solve 1D two-layer shallow water equations. This numerical model is intended to describe two superposed layers of immiscible fluids flows which differ in velocity, thickness and density. The system contains source terms due to bottom topography, and nonconservative products describing momentum exchange between the layers. Their presence makes the system nonconservative, and eventually non hyperbolic. To avoid the discretization of the term source, the continuous equations are rewritten in a non-conservative form and discretized using nodal polynomial basis functions of arbitrary order in space on each element of an unstructured computational domain. To complete the discretization in space, we choose the numerical flux based in the local Lax-Friedrichs flux. A third-order explicit Runge-Kutta scheme is used to advance the solution in time. In spite of the local time steps the scheme is locally conservative, fully explicit, and arbitrary order accurate in space and time for transient calculations. The versatility of the approach is illustrated on a number of numerical examples, in which we successfully capture (quasi) steady-state solutions and propagating interfaces. The numerical scheme is also applied to a realistic simulation of the flow through the Strait of Gibraltar: Real bathymetric and coast-line data are considered to include in the model the main features of the abrupt geometry of this natural strait connecting the Atlantic Ocean and the Mediterranean Sea. Initially a steady-state solution is obtained from a "lock- exchange" experiment. Then we use this solution to simulate a tidal experiment in the Strait where the main semidiurnal and diurnal tides are imposed as boundary conditions.

REFERENCES

- [1] F. Bouchut and T. Morales de Luna, An entropy satisfying scheme for two-layer shallow water equations with uncoupled treatment. *M2AN* 42 683698 (2008), pp. 683698.
- [2] A. Kurganov and G. Petrova, Central-Upwind Schemes for Two-Layer Shallow Water Equations.
- [3] Hesthaven JS, Warburton T (2002) High-order nodal methods on unstructured grids. I. Time-domain solution of Maxwells equations. *J Comp Phys* 181(1):186221.

*Joint work with M. SEAID, M. WAKRIM and F.BENKHALDOUN.

Optimality conditions and duality in nondifferentiable multiobjective programming

Izhar Ahmad *

Abstract

A nondifferentiable multiobjective problem is considered. Fritz John and Kuhn-Tucker type necessary and sufficient conditions are derived for a weak efficient solution. Kuhn-Tucker type necessary conditions are also obtained for a properly efficient solution. Weak and strong duality theorems are established for a Mond-Weir type dual. Moreover, for a converse duality theorem we discuss a special case of nondifferentiable multiobjective problem, where subgradients can be computed explicitly.

*Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran-31261, Saudi Arabia Email: drizhar@kfupm.edu.sa.

Backward error in linear least squares problems: estimates, their accuracy, and implementation

Pavel Jiránek *

Abstract

We consider a linear least squares (LS) problem: find $\hat{x} \in \mathbb{R}^n$ such that $\|b - A\hat{x}\|_2 = \min_{x \in \mathbb{R}^n} \|b - Ax\|_2$, where $A \in \mathbb{R}^{m \times n}$ is a given matrix and $b \in \mathbb{R}^m$ is a right-hand side vector. We are interested in computing the backward error for a given approximation x to the LS solution \hat{x} . It was shown in [3] that the backward error μ defined by $\mu \equiv \min_{E, f} \{ \| [E, \theta f] \|_F; (A+E)^T(A+E)x = (A+E)^T(b+f) \}$ is given by $\mu = \min\{\omega, \sigma_{\min}(M)\}$, where $\theta > 0$ is a given weighting parameter, $\omega \equiv \min_{E, f} \{ \| [E, \theta f] \|_F; (A+E)x = b+f \} = \theta \|r\|_2 / \sqrt{1 + \theta^2 \|x\|_2^2}$ is the backward error associated with x in the linear equations $Ax = b$, $M \equiv [A; \omega(I - rr^\dagger)]^T$, and $r \equiv b - Ax$ is the residual vector. The minimal singular value of the matrix M can be expensive to compute and some estimates of μ were proposed by Stewart [1] and Karlson and Waldén [2]. We analyze their accuracy and show that the Stewart's bounds are good approximations of μ provided the quantity ω lies outside the interval defined by the extremal singular values of the matrix A . On the other hand, we extend the existing analysis of the Karlson-Waldén's estimate and prove that it is always a good approximation to the LS backward error. We also discuss their implementation in the LSQR method of Paige and Saunders.

REFERENCES

- [1] G. W. Stewart. *Research, development, and LINPACK*. In *Mathematical Software III*, pages 1–14. Academic Press, New York, 1977.
- [2] R. Karlson and B. Waldén. *Estimation of optimal backward perturbation bounds for the linear least squares problem*. BIT, 37(4):862–869, 1997.
- [3] B. Waldén, R. Karlson, and J.-G. Sun. *Optimal backward perturbation bounds for the linear least squares problem*. Numer. Linear Algebra Appl., 2(3):271–286, 1995.

*CERFACS, 42 Avenue Gaspard Coriolis, 31057 Toulouse, France, E-mail: jirane@cerfacs.fr. Joint work with Serge Gratton (INPT-IRIT, ENSEEIHT, 2 Rue Camichel, BP 7122, 31071 Toulouse, France, E-mail: serge.gratton@enseeiht.fr) and David Titley-Peloquin (Mathematical Institute, University of Oxford, 24-29 St-Giles', Oxford, OX1 3LB, UK, E-mail: david.titley-peloquin@maths.ox.ac.uk).

Approach HUM for regional controllability of semilinear parabolic systems : state and gradient

Asmae Kamal *

Abstract

The problem of regional controllability for semi-linear parabolic systems is considered. We show how one can reach a desired state or gradient of state given only on a part of the system domain. The proposed approach combines the extension of the Hilbert Uniqueness Method (HUM) and the fixed point techniques, leading to an algorithm which is successfully performed through numerical examples.

Keywords: Semilinear systems - Parabolic systems - Regional - Controllability - Gradient - Fixed point theorem.

REFERENCES

- [1] Brezis H., *Analyse fonctionnelle : théorie et application*. Masson. 1983.
- [2] Fabre C., Puel J. P. and Zuazua E., Approximate controllability of the semilinear heat equation, *Proc. Royal Soc. Edinburgh*, 125 A, 1995, pp. 31-61.
- [3] Lions J.L. and Magenes E., *Problèmes aux limites non homogènes et applications*. Vol. 1 et 2. Dunod. 1968.
- [4] Lions J.L., *Contrôlabilité exacte, Perturbations et stabilisation des systèmes distribués*. Tome 1. Masson. 1988.
- [5] Zerrik E., Boutoulout A. and Kamal A., *Regional gradient controllability for parabolic systems*, *Int. J. Appl. Math. Comp. Sci.*, Vol. 9, No. 4, pp. 767-787, 1999.
- [6] Zerrik E. and Kamal A., *Output controllability for semi-linear distributed parabolic systems*, *Journal of Dynamical and Control Systems*, Vol. 13, No. 2, April 2007, pp. 289-306.

*Joint work with Ali Boutoulout and Sidi Ahmed Beinane

TSI Team, Moulay Ismail University, Faculty of Sciences, Meknes, Morocco.

E-mail : as2.kamal@yahoo.fr, boutouloutali@yahoo.fr and beinane06@hotmail.com.

Approximation of curves and/or surfaces by ODE variational splines

Abdelouahed Kouibia *

Abstract

In Geology, Geophysic and other Earth Sciences, it is usual to find the construction problem of curves and surfaces from a Lagrange or Hermite data set. It is convenient that such curve or surface verifies some physical or geometrical conditions such as preserving shape, some fairness criteria (see for example [2]).

Likewise, in CAD and industrial design, the construction of curves and/or surfaces contours are done traditionally by using geometric primitives such as lines, conic and others which can be characterized by some simple equations. However, in many problems of engineering, architecture, geology and others, it is needed smooth curves and/or surfaces whose shapes cannot be described by a simple equation. Particularly, this problem appears in the automotive or aerospace industries where the section of manufactured objects are designed from some interpolation or approximation data, and also verifying some hydrodynamic properties that can be modeled by certain ODE. Hence, it is natural to use free-form curves. In [1] the authors present a design method for free-form curves from a set of approximation points and a boundary value problem for an ODE.

This paper deals with a construction problem of free-form curves and/or surfaces from data constituted by some approximation points and a boundary problem for an ODE. The solution of this problem is called an ODE variational spline. We discretize the problem in a suitable finite dimensional space. We study the existence and uniqueness of the solution of such problem. Then, we establish some convergence and error estimations results. Finally, we analyze some numerical and graphical examples to show the validity and effectiveness of our method.

REFERENCES

- [1] A. Kouibia, M. Pasadas and M.L. Rodriguez, *Construction of ODE curves*, Numerical Algorithms **34** (2003) 367-377.
- [2] A. Kouibia and M. Pasadas, *Approximation of surfaces by fairness bicubic splines*, Advances in Computational Mathematics **20** (2004) 87-103.

M. Pasadas, Z. Belhaj** and K. Najib**

*: Dept of Applied Math., Faculty of Sciences, C.P. 18071, University of Granada.

kouibia(mpasadas)@ugr.es

** : Ecole Nationale de l'Industrie Minérale, Rabat, B.P.753 Rabat-Agdal, MAROC.

najibkhalid(z.belhaj)@gmail.com

Purely algebraic domain decomposition methods for the incompressible Navier-Stokes equations

Pawan Kumar *

Abstract

In the context of domain decomposition methods, an algebraic approximation of the transmission condition (TC) is proposed in [2, 3, 1]. For the case of non overlapping domains, approximation to the TCs are analogous to the approximation of the Schur complements (SC) in the incomplete block factorization. The basic idea is to approximate the SC by a small SC approximations in patches. The computation of these local transmissions are constructed independently, thus enhancing the parallelism in the overall approximation.

In this work, a new computation of local Schur complement is proposed and the method is tested on incompressible Navier-Stokes problems. The earlier attempts used in the literature approximate the TC by building small patches around each node. We generalize the method by aggregating the nodes and thus reducing the overlapping computation of local TCs. Additionally, the approach of aggregating the nodes is based on the “numbering” of the nodes rather than on the “edge connectivity” between the nodes.

With the new aggregation scheme, the construction time is significantly less. Furthermore, the new aggregation based approximation leads to a completely parallel solve phase. The new method is tested on the difficult cavity problem with high reynolds number on uniform and stretched grid. The parallelism of the new method is also discussed.

References

- [1] F. Nataf, H. Xiang, V. Dolean. *A Coarse Space Construction Based on Local DtN Maps*, 2010, <http://hal.archives-ouvertes.fr/hal-00491919/fr/>.
- [2] F.-X. Roux, F. Magoulès, L. Series, Y. Boubendir, Algebraic approximation of Dirichlet-to-Neumann maps for the equations of linear elasticity, *Comput. Methods Appl. Mech. Engrg.*, 195, 2006, 3742-3759
- [3] F.-X. Roux, F. Magoulès, L. Series, Y. Boubendir, Approximation of Optimal Interface Boundary Conditions for Two-Lagrange Multiplier FETI Method, *LNCSE*, volume 40, 2005, 283-290

*Institut Henri Poincare (IHP) - UMS 839 (CNRS/UPMC) 11, rue Pierre et Marie Curie - 75231 Paris Cedex 05 France: pawan.kumar@u-psud.fr

The work was completed while the author was a participant at Institut Henri Poincare, Paris in the trimester program control and PDEs during October 2010 to December 2010.

Adaptation of the M. N. El Tarazi theorem and application to the variable parallel overlapping domain decomposition method

Mohamed Laaraj *

Abstract

It's very classic to associate to a Schwarz alternating method a property of contraction with respect to the uniform weighted norm $\| \cdot \|_{e,\infty}$, where e is the positive eigenfunction associated with the smallest eigenvalue of the operator.

We present here one parallel overlapping domain decomposition with method based on two decompositions with domains and without overlapping, where the first ones serve to define the operations of restriction and the second ones serve to the resolution.

We shall also present an adaptation of the M. N. El Tarazi theorem. This is going to assure us the convergence of the parallel asynchronous iterations with multiple initializations associated to the overlapping domain decomposition which can vary from an iteration to the other one

It is worthwhile to note that this kind of property can be used in order to obtain a dynamic load balancing during the run of asynchronous iterations.

REFERENCES

- [1] J. C. Miellou, M. Laaraj, M. Gander, *Overlapping multisubdomains asynchronous fixed point methods for elliptic boundary value problems*, VSP International Science Publishers Utrecht, The Netherland, Tokyo (1999) 261-266.
- [2] M. Laaraj, *Equations de point fixe associées à une décomposition en sous domaines multiples, pour des problèmes elliptiques et paraboliques*, Thèse de l'Université de Franche-Comté (2000).
- [3] M. Charnay, *Itérations chaotiques sur un produit d'espaces métriques*, C. r. Acad. Sci., Paris, Ser. A279 (1974) 205-208.

*Université Hassan II Mohammadia. Maroc. E-Mail : laaraj@yahoo.com

On an algebraic optimized Schwarz preconditioning: performance and applications

Lahcen Laayouni*

Abstract

In this paper we investigate the performance of the algebraic optimized Schwarz methods. These methods are based on the modification of the transmission block matrices between subdomains. The transmission blocks are replaced by new blocks to improve the convergence of the corresponding algorithms. In the optimal case, convergence in two iterations can be achieved. We are interested in how the algebraic optimized Schwarz methods perform as preconditioners solving differential equations. We are also interested in their asymptotic behavior with respect to change in problems parameters. We present different numerical simulations corresponding to different type of problems in two- and three-dimensions.

*School of Science and Engineering, Al Akhawayn University, Avenue Hassan II, 53000 P.O. Box 1630, Ifrane, Morocco (L.Laayouni@au.ma).

Joint work with Daniel Szyld: Department of Mathematics, Temple University (038-16), 1805 N. Broad Street, Philadelphia, Pennsylvania 19122-6094, USA (szyld@temple.edu)

Nonmonotone spectral projected gradient methods applying to Thomson problem

Halima Lakhbab*

Abstract

The problem of finding how electrons optimally distribute themselves on the sphere is a well-known and difficult one. It is ranked 7 in Stephen Smale's famous list of 18 unsolved mathematical problems to be solved in the 21st century.

The original question was posed by J.J. Thomson [2] after his discovery of the electron in 1897. Thomson conjectured that the knowledge of the positions of the electrons inside the atoms is essential to understanding the regularity of the chemical elements in the periodic table. Yet the problem turned out to be important for many fields, from biology to telecommunications. Thomson's problem is consisting in distributing a number N of equal charges on a sphere. In our study we modelize this problem by minimizing the potential energy of N particles on sphere, and we apply Nonmonotone Spectral Projected Gradient Methods (NSPG) [1] to find local minimums, and we give numerical results for various number N of points. We intend afterward to combine the NSPG algorithm with metaheuristics to approach global minimums.

REFERENCES

- [1] E.G.Birgin, J.M.Martínez, M.Raydan , *Nonmonotone Spectral Projected Gradient Methods on Convex Sets*, SIAM Journal on Optimization (2000) Volume 10 Issue 4, .
- [2] J.J. Thomson, *On the Structure of the Atom.*, Philosophical Magazine Series 6, Vol 7 (1904) 237 – 265.

*Joint work S. El Bernoussi and A. El Harif; Facult des scieees, Rabat.

Mathematical analysis of a system modeling ions electromigration through biological membranes

Hamid Lefraich *

Abstract

In this work we present the mathematical analysis of a system modeling ion migration through biological membranes. The model includes both the effects of biochemical reaction between ions and fixed charges. The model is a nonlinear coupled system. In the first we describe the mathematical model. To develop the mathematical analysis of our model, we define an approximating scheme and by using Schauder fixed point theorem in ordered Banach spaces, we show the existence of a solution for this approached problem. Finally by making some estimations we prove that the solution of the truncated system converge to the solution of our problem.

REFERENCES

- [1] A. D. Macgillivray, *Nernst-Planck equations and the electroneutrality and Donnan equilibrium assumptions*, journal of chemical physics (1968) 2903-2907.
- [2] J. Henry, B, Louro *asymptotic analysis of reaction-diffusion-electromigration system*, rapport INRIA 2048(1993).
- [3] N. Alaa, N. Idrissi Fatmi, J. R. Roche *Mathematical Analysis for a model of Nickel-Iron alloy electrodeposition on rotating disk electrode: parabolic case*, International Journal of Mathematics and Statistics, Spring 2008, Volume 2, Number S08(2008)30-48.

*Joint work with N. Alaa, Laboratoire LAMAI Faculte des Sciences et Techniques de Marrakech

Time domain decomposition with time reversible integration for initial value problem

Patrice Linel *

Abstract

This paper concerns the development of a parallel method to solve ODE. This kind of systems are involved in most of the modeling as mechanical systems modeling or chemical reactor modeling. These systems are usually nonlinear with a large number of unknowns and a large computational cost.

We propose a time domain decomposition method based on the Schwarz algorithm that breaks the sequentiality of the integration scheme for system of ODEs[4]. For system of linear ODEs(Helmholtz equations for example), the algorithm shows a linear divergence or convergence allowing to apply the Aitken's acceleration of the convergence technique[1] to obtain the solution at the boundaries of the subdomains. For nonlinear problems, we apply some acceleration of the convergence of nonlinear sequences algorithms[3] to the domain decomposition methods.

A second approach is developed using time-reversible integration scheme and a system of conditions satisfied by the subdomains's solution. In case of nonlinear system of ODEs, the system of constraints is solved by a Newton method[2].

Implementation, numerical results and efficiency will be discussed and illustrated by examples.

REFERENCES

- [1] Garbey M. and Tromeur-Dervout D., *On some Aitken-like acceleration of the Schwarz method*, International Journal for Numerical Methods in Fluid (2002) 1493-1513.
- [2] Deuffhard Peter, *Newton methods for nonlinear problems*, Springer Series in Computational Mathematics (2004).
- [3] Brezinski C., *Convergence acceleration during the 20th century*, Journal of Computational and Applied Mathematics (2000) 1-21.
- [4] Linel P. and Tromeur-Dervout D., *Aitken-Schwarz and Schur complement methods for time domain decomposition* Advances in Parallel Computing

*Joint work with D.Tromeur-Dervout, Université de Lyon, Université Lyon1, CNRS, ICJ-UMR5208

A dual decomposition method for color image restoration

Cédric Loosli *

Abstract

We study a dual decomposition method for a weighted total variation regularization term for 3D vectors with L^1 -norm as data fidelity term. By introducing a suitable auxiliary unknown, the unconstrained minimization problem is transformed into a saddle-point problem. Applying a Uzawa block relaxation method, we obtain an iterative algorithm in which each component of the unknown is treated independently.

Let Ω be a bounded domain in \mathbb{R}^2 . A color image can be interpreted as a vector-valued function $u = (u_r, u_g, u_b)$ defined on Ω , where r , g and b stand for RGB channels. We define the weighted total variation for 3D vectors by

$$J(u) = \int_{\Omega} \sqrt{|\nabla u_r|^2 + |\nabla u_g|^2 + |\nabla u_b|^2} dx, \quad (1)$$

where $|\cdot|$ stands for Euclidean norm. Given a noisy image $f = (f_r, f_g, f_b)$, the original image u can be recovered through the minimization problem

$$\min_u E(u) = J(u) + F(u) \quad (2)$$

where

$$F(u) = \lambda \int_{\Omega} |u - f| dx$$

is the (L^1) fidelity term. By introducing the auxiliary unknown $p = f - u$ we replace the unconstrained minimization problem (2) by the constrained minimization problem

$$\min_{(u,p) \in K} E(u, p) = J(u) + F(p), \quad (3)$$

where $K = \{(u, p) \in X \times X \mid u + p - f = 0 \text{ in } X\}$. We then apply a Uzawa block relaxation algorithm to the corresponding augmented Lagrangian to obtain a fast iterative restoration algorithm using Chambolle scheme [1] and explicit calculations (for p) in every iteration, using results of [2] on gray-scale images.

REFERENCES

- [1] Chambolle A., *An algorithm for total variation minimization and applications*, J. Math. Imaging Vision. 20 (2004), 89–97.
- [2] Koko J. and Jehan-Besson S., *An augmented Lagrangian method for $TV_g + L^1$ -norm minimization*, J. Math. Imaging Vision 38 (2010), 182–196.

*Joint work Jonas Koko and Jehan-Besson, LIMOS, Université Blaise-Pascal – CNRS UMR 6158, Clermont-Ferrand, France

On semifields of order q^4 : a computational approach

Mashhour Al-Ali Bani Ata *

Abstract

Semifields are an interesting subject of study in Mathematics and have applications in engineering areas such as coding theory and cryptography, in particular in the problem of linear and non-linear secure net work coding. Recently finite projective plane geometry and in particular semifields play a significant role in optimization techniques for structural design and automated techniques for finite element modeling, also in the organization for records with multiple-valued attributes. This talk is mainly based on linear algebra techniques and computational methods related to eigenvalue problems to investigate semifields of order q^4 over the Galois field F_q admitting a Klein 4-group of automorphisms and having finite Galois fields $GF(q^2)$ in their left nuclei. Some applications will be mentioned.

*Department of Mathematics Faculty of Basic Education PAAET, Shamiya, Kuwait e-mail:mashhour ibrahim@yahoo.com

A fast algorithm for solving Toeplitz plus Hankel structured total least squares problems

Nicola Mastronardi *

Abstract

The Structured Total Least Squares (STLS) problem is a natural extension of the Total Least Squares (TLS) problem when constraints need to be imposed on the matrix structure of the overdetermined linear system to be solved. Similar to the ordinary TLS approach, the STLS approach can be used to determine the parameter vector of a linear model, given some noisy measurements. In many signal processing applications, the imposition of this matrix structure constraint is necessary for obtaining Maximum Likelihood (ML) estimates of the parameter vector. In this talk we consider the Toeplitz plus Hankel STLS problem, i.e. an STLS problem in which the Toeplitz plus Hankel structure needs to be preserved.

A fast implementation of a numerical method, based on the generalized Schur algorithm [2, 3], for solving this frequently occurring STLS problem is proposed.

The performance of the latter algorithm is compared to the one of another Toeplitz plus Hankel STLS algorithm recently proposed [2], and applied to estimate the frequencies of signals made by the sum of sinusoids.

REFERENCES

- [1] F.K.W. Chan, H.C. So, W.H. Lau and C.F.Chan, *Structured total least squares approach for efficient frequency estimation*, Signal Processing 91 (2011) 1043-1047.
- [2] T. Kailath and A. Sayed, *Displacement Structure: Theory and Applications*, SIAM Review, 32 (1995) 297-386.
- [3] P. Lemmerling, N. Mastronardi and S. Van Huffel, *Fast algorithm for solving the Hankel/Toeplitz structured total least squares problem*, Numer. Algorithms 23 (2000) 371-392.

*IAC-CNR, Bari, Italy. Joint work with Teresa Laudadio, IAC-CNR, Bari, Italy, and Sabine Van Huffel, Department of Electrical Engineering, Katholieke Universiteit Leuven, Leuven, Belgium

Stopping rules for iterative methods in nonnegatively constrained deconvolution

Ornella Menchi *

Abstract

An interesting inverse problem is the two-dimensional discrete deconvolution, whose goal is to reconstruct an object \mathbf{x}^* from its image \mathbf{b} . In addition to the blurring introduced by the optical system, represented by a matrix A , \mathbf{b} is affected by a noise η , assumed to be ruled by uncorrelated Poisson and Gaussian statistics. The model for this problem is $A\mathbf{x}^* = \mathbf{b} - \eta$, where A , \mathbf{x}^* and \mathbf{b} are nonnegative. Because of the ill-determined rank of A and the presence of the noise, regularization is required, coupled with strategies for enforcing nonnegativity. When the large size of the problem prevents regularization by filtering, an iterative regularization method is required. In [2] the performance of some methods, belonging to the class of Scaled Gradient Projection methods, has been analyzed from various points of view. We consider here three of them, which have been found the most efficient ones for the reconstruction accuracy and the convergence speed, namely EM, SGP and WMRNSD. These three methods enjoy the semiconvergence property, i.e. the computed iterations $\mathbf{x}^{(k)}$ first approach \mathbf{x}^* , then go away and the choice of the index K at which the iteration should be stopped is critical. Ideally, the iteration should be stopped when the solution error $\epsilon^{(k)} = \|\mathbf{x}^{(k)} - \mathbf{x}^*\|$ is minimum. Our aim is to compare the behavior of three widely used stopping rules, GCV, UPRE and the discrepancy principle, with the selected methods. GCV and UPRE have been designed originally for regularization methods whose influence matrix is explicitly known, then they have been generalized [1, 3] for iterative methods through the use of the Trace Lemma.

REFERENCES

- [1] J.M. Bardsley, *Stopping Rules for a Nonnegatively Constrained Iterative Method for Ill-Posed Poisson Imaging Problems*, BIT, **46**, (2008) 651-664.
- [2] P. Favati, G. Lotti, O. Menchi and F. Romani, *Performance analysis of maximum likelihood methods for regularization problems with nonnegativity constraints*, Inverse Problems, **26**,(2010) 085013.
- [3] K.M. Perry and S.J. Reeves, *A Practical Stopping Rule for Iterative Signal Restoration*, IEEE Transaction on Signal Processing, **42**, (1994) 1829-1933.

*Department of Computer Science, University of Pisa.

Joint work with P. Favati, IIT-CNR Pisa, G. Lotti, Department of Mathematics, University of Parma and F. Romani, Department of Computer Science, University of Pisa.

Inner-iteration GMRES methods for underdetermined least squares problems

Keiichi Morikuni *

Abstract

Inner-iteration preconditioners combined with the GMRES methods are proposed for solving underdetermined least squares problems

$$\min_{\mathbf{x} \in \mathbf{R}^n} \|\mathbf{b} - A\mathbf{x}\|_2, \quad (1)$$

where $A \in \mathbf{R}^{m \times n}$, $\mathbf{b} \in \mathbf{R}^m$, and $m < n$.

We can precondition (1) from the right

$$\min_{\mathbf{u} \in \mathbf{R}^n} \|AB\mathbf{u} - \mathbf{b}\|_2, \quad \mathbf{x} = B\mathbf{u} \quad (2)$$

or from the left

$$\min_{\mathbf{x} \in \mathbf{R}^n} \|B\mathbf{b} - BA\mathbf{x}\|_2 \quad (3)$$

by using a preconditioner $B \in \mathbf{R}^{n \times m}$. The results in [2] gave conditions for B for the convergence of GMRES. However, as noted in [1], when solving inconsistent systems ($\mathbf{b} \notin \mathcal{R}(A)$), the effective condition number becomes dangerously large. Hence, GMRES for (2) will practically breakdown before it determines a least squares solution.

On the other hand, we show that (3) can be consistent when $\mathcal{R}(B^T) = \mathcal{R}(A)$. Thus, GMRES can practically determine a least squares solution for (1) even if $m < n$ and $\mathbf{b} \notin \mathcal{R}(A)$. To form such a preconditioner B , we propose using inner-iteration preconditioners. The inner iterations do not require a preconditioning matrix and can save storage memory.

Numerical experiments illustrate that the methods are efficient and robust for large ill-conditioned and rank-deficient problems, outperforming previous methods.

REFERENCES

- [1] P. N. Brown, and H. F. Walker, *GMRES on (nearly) singular systems*, SIAM J. Matrix Anal. Appl., 18 (1997), 37–51.
- [2] K. Hayami, and J.-F. Yin, and T. Ito, *GMRES methods for least squares problems*, SIAM J. Matrix Anal. Appl., 31 (2010), 2400–2430.

*The Graduate University for Advanced Studies, Sokendai, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo, 101-8430, Japan.

Joint work with Ken Hayami, National Institute of Informatics, 2-1-2, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan.

Identification of Priestley-Taylor transpiration parameters used in TSEB model by genetic algorithm method over olive irrigated area.

Abdelhaq Mouida *

Abstract

The accuracy degree of extracted canopy latent heat from canopy net radiation is depending extremely to the proposed Priestley-Taylor approximation. This extracting canopy latent heat is an initial approximation to compute iteratively partitioned energy components to soil and vegetation using in Two Source Energy Balance (TSEB) Model. This approximation is using a Priestley-Taylor coefficient (ap) and fractional of Leaf Area Index (fg) that is green. These standard values (ap) and (fg), are empirically estimated. Many studies forced to proceed empirically, and asked whether it was still a principal component of evaporation from a wet region. They looked that a value of coefficient (ap= 1.26) was found to fit data from several sources especially for wet regions. The TSEB Model uses either this formula adding an other coefficient (fg=1) which is a fractional of Leaf Area Index that is green (Norman et al, 1995; Kustas et al 1999). These studies are also proposed values of (ap) and (fg) ranging respectively from 0.5 to 3 and 0 up to 1 (Lhomme et al 1996; McNaughton et al 1989); This study is focused to identify these two transpiration parameters (ap) and (fg) by Genetic Algorithm method to accurately predict patterns of turbulent energy fluxes by TSEB Model (Norman et al. 1995), over irrigated olive orchard in semi-arid area (Marrakech, Morocco). The (ap) and (fg) are depending on local climatic characteristics and data measurements accuracy for different periods of the year 2003. In this work, for a semi-arid areas, we suggest to use stochastic method as Genetic algorithms (GAs) to identify Priestley-Taylor transpiration Parameters over olive irrigated area (in wet and dry conditions). GAs approach are used for solving parameters estimation for its independency to problem types, such as non linear, multimodal and/or non-differentiable functions (Goldberg, David E, 1989). GAs are a way of addressing hard search and optimization problems which provides a good solution although it requires large execution time.

REFERENCES

- [1] Norman et al.1995, *Source approach for estimating soil and vegetation energy fluxes in observations of directional radiometric surface temperature; Agricultural and Forest* , Meteorology 77 (1995) 263-293.
- [2] Lhomme et al. 1996, *A Theoretical basis for the Priestley-Taylor coefficient;* ORSTOM, Laboratoire dHydrologie, B.P, 5045, 34032 Montpellier; France.
- [3] Goldberg et al. 1989, *Genetic Algorithms in Search, Optimization and Machine Learning*
- [4] McNaughton et al 1989, *An evaluation of the Priestley and Taylor equation and the complementary relationship using results from a mixed-layer model of the convective boundary layer*, Spriggs Plant Physiology Division, D.S.I.R. Private Bag, Palmerston North New Zealand.

*Marocmeteo, (DMN) Direction Régionale Météorologique Nord 21 Bis, Rue Ouled Jerrar Lot Courdess, Souissi Rabat, Morocco.

Joint work Nour Eddine ALAA, Faculty of Science and Techniques Laboratory LAMAI, University of Cadi Ayad Marrakech, Morocco, alaanouredine @gmail.com

A enhanced combined genetic algorithm-fuzzy logic controller (GA-FLC): application to free boundary problem

Mourad Nachaoui*

Abstract

This work presents an enhanced combined genetic algorithm-fuzzy logic method to solve a free boundary problem of welding process. In standard Genetic algorithms, the upper and lower limits of the search regions should be given by the decision maker in advance to the optimization process. In general, a large search region is used in fear of missing the global optimum outside the search region. Therefore, if the search region is able to adapt toward a promising area during the optimization process, the performance of GA will be enhanced greatly. A combined genetic algorithm with a fuzzy logic controller. This controller monitors the variation of the design variables during the first run of the genetic algorithm and modifies the initial bounding intervals to restart a second round of the genetic algorithm. A new defuzzification method is proposed to perform a fuzzy logic controller. Compared to previous works using classical Genetic algorithms, our method proved to be more efficient in computation time and accuracy of the final solution.

REFERENCES

- [1] Chakib, A; Ellabib, A; Nachaoui, A; Nachaoui, M. *A shape optimization formulation of weld pool determination.*, Submitted.
- [2] Chakib, A; Nachaoui, A; Nachaoui, M., Shikhliinskaya R. Y. *hybrid defuzzification method for fuzzy controllers*, submitted.

*Joint work with A. Chakib, Laboratoire de Mathématiques et Applications Université Sultane Moulay Slimane, Faculté des Sciences et Techniques, B.P.523, Béni-Mellal, Maroc and A. Nachaoui, Laboratoire de Mathématiques Jean Leray CNRS / Université de Nantes UMR6629 2 rue de la Houssinière, BP92208 44322 Nantes, Laboratoire de Mathématiques et Applications, université Sultane Moulay Slimane.

Preconditioning of systems arising from finite element discretizations of phase-field models

Maya Neytcheva *

Abstract

Flows dominated by capillarity and wetting are important in many processes in nature and are of increasing interest, in particular, in microfluidic applications. Accurate and computationally efficient numerical simulations of such flows remain a challenging task and put extra demands on the numerical solution methods used.

In this talk we consider preconditioned iterative solution methods to solve the algebraic systems of equations arising from finite element discretizations of multiphase flow problems, modeled using the phase-field model, coupled with the Navier-Stokes equation. The model allows us to simulate the motion of a free surface in the presence of surface tension and surface chemistry energy.

We focus on the phase-field model, described by the Cahn-Hilliard equation. The problem is time-dependent and nonlinear. We consider the task to solve the linear systems with the arising Jacobian matrices by a preconditioned iterative solution method. When discretized, the Cahn-Hilliard equation gives rise to large scale algebraic systems of equations with matrices of a particular two-by-two block form. The block structure is utilized when constructing preconditioners to be used in an iterative procedure. Interestingly enough, the preconditioning techniques are applicable to solving systems with symmetric complex matrices as well as for problems arising from some optimization problems with a constraint given by a partial differential equation.

We illustrate the performance of the preconditioners with numerical experiments.

REFERENCES

- [1] O. Axelsson, M. Neytcheva, *Operator splittings for solving nonlinear, coupled multiphysics problems with an application to the numerical solution of an interface problem*, in progress, 2011.
- [2] P. Boyanova, M. Do-Quang, M. Neytcheva, *Solution methods for the Cahn-Hilliard equation discretized by conforming and non-conforming finite elements*, in progress, 2011,

*Department of Information Technology, Uppsala University, Sweden. Joint work with Minh Do-Quang, Linné Flow Centre, Department of Mechanics, Royal Institute of Technology, Stockholm, Sweden

Generalized circulant preconditioners for Toeplitz systems

Silvia Noschese *

Abstract

An $\{e^{i\varphi}\}$ -circulant is a generalized circulant matrix whose columns are obtained by multiplying the first entry of a circular shift of the preceding column by $e^{i\varphi}$. Matrix-vector products with generalized circulants can be evaluated efficiently with the aid of the Fast Fourier Transform (FFT) algorithm, similarly as matrix-vector products with circulant matrices. This makes the application of generalized circulants as preconditioners for linear systems of equations with a Toeplitz matrix attractive. We describe several ways to determine generalized circulant preconditioners and show convergence properties of preconditioned conjugate gradient iteration. Numerical results illustrate that generalized circulant preconditioners can give faster convergence than circulant preconditioners.

*Math. Univ. Roma La Sapienza, Italy. Joint work with Lothar Reichel, Kent State University, USA.

A nonlinear four-point subdivision scheme

Otheman Nouisser *

Abstract

Subdivision schemes are efficient methods for generating curves and surfaces from discrete sets of control points. The important schemes for applications are schemes for surfaces, yet schemes generating curves constitute a basic tool for the design, study, and understanding of schemes generating surfaces.

In this paper, we propose to study the scheme

$$(Sf)_{2i} = \frac{3}{4}f_i + \frac{1}{4}f_{i+1} - \frac{3}{32}M(\Delta^2 f_{i-1}, \Delta^2 f_i)$$
$$(Sf)_{2i} = \frac{1}{4}f_i + \frac{3}{4}f_{i+1} - \frac{3}{32}M(\Delta^2 f_i, \Delta^2 f_{i-1})$$

with any M that is originally defined as a positive-valued function for positive arguments and is extended to the whole of \mathbb{R}^2 by setting $M(x, y) = -M(|x|, |y|)$ if $x < 0, y < 0$ and $M(x, y) = 0$ if $xy \leq 0$.

We study analytic properties, such as convexity preservation, convergence, smoothness of the limit function, stability and approximation order.

REFERENCES

- [1] N. Dyn, D. Levin and J.A. Gregory, *A 4-point interpolatory subdivision scheme for curve design*, Comput. Aided Geom. Design 4 (1987) 257-268.
- [2] M. Floater and C. Michelli, *Nonlinear stationary subdivision*, in Approximation Theory: In Memory of A.K. Varma, N.K. Govil et al. (Eds.) Marcel Dekker, New-york, (1998) 209-224.
- [3] F. Kuijt, *Convexity preserving interpolation-stationary nonlinear subdivision and splines*, Thesis University of Twente, (1998).

*Faculté Polydisciplinaire de Safi, Route sidi-Bouzide, 46 000 Safi, Maroc.

Mathematical techniques for image retoration

Fatima Zohra Nouri *

Abstract

Many techniques have been used for image processing, such as restoration, contour detection, segmentation ect... Among these techniques, partial differential equations (PDEs) have been our main interest, in particular second order ones Nouri et al (2006) and fourth order PDEs, Maoumi et al (2008). Image inpainting is a fondamental problem in image processing and has many applications Bertozzi et al (2007) and Chan et al (2006) motivated by the recent tight frame based method on image restoration in either the image or the transform domain. In this work, we present an inpainting model based on non-Newtonian fluids due to Igehy et al (1997), for damaged wavelet coefficients. The advantage of this model is to make a benefit from the smoothing model and correct the lying out of the contours by putting them more clearly. Numerical results show that better inpainting quality can be achieved with much less computing time, compare to the model by Hadji et al (2010).

REFERENCES

- [1] A. Bertozzi, S. Esedoglu, and A. Gillette, *Inpainting of binary images using the Cahn-Hilliard equation*, IEEE Transactions on Image Processing, vol. 16, no. 1, January 2007.
- [2] T. F. Chan, J. Shen, and H.-M. Zhou, *Total variation wavelet inpainting*, Journal of Mathematical Imaging and Vision, vol. 25, no. 1, July 2006.
- [3] M.L. Hadji, M. Maoumi and F.Z. Nouri, *A Wavelet Inpainting by a Toxitrop Model*, iv, 2010 14th International Conference Information Visualisation, 2010, IEEE ISBN: 978-0-7695-4165-5, doi.ieeecomputersociety.org/10.1109/IV.2010.96, pp. 559-563.
- [4] M.Maoumi and F.Z. Nouri , *Image Restoration by Partial Differential Equations*, IEEE GMAI06 Proceedings of the conference on Geometric Modeling and Imaging: New Trends ISBN:0-7695-2604-7, 2006, pp 250 254.
- [5] M. Maouni , F.Z. Nouri and D. Meskine, *Image Restoration by Non-standard Diffusion*, IEEE Geometric Modeling and Imaging Modern Techniques and Applications Volume-Issue 09-11, July 2008, pp 98-101.

*Joint work with M.L. Hadji, M. Maoumi; Laboratoire de Mathématiques Appliquées et LIR- IMA, Université Badji Mokhtar, Annaba (Algérie)

Nearest stable polynomial using successive convex approximations

Francois-Xavier Orban de Xivry *

Abstract

The stability of systems has always been a central question in the system and control area. The literature on the subject is abundant and still growing. Finding the smallest perturbation to make a stable system unstable has been a well explored problem. However, to the best of our knowledge, the question of finding a nearest stable system to an unstable one is still open. In this paper, our interest goes to the case of polynomials which can be used to represent systems, for example, in signal processing and other domains. The goal is to obtain a locally optimal solution in a reasonable computational time. We build our theory in the general framework of stable matrices but will specially emphasize the case of polynomials, given its simplicity.

In order to achieve a moderate complexity, we need to avoid the use of general optimization scheme such as interior point methods. Rather we write the first optimality conditions of our problem and solve them to obtain a close form solution, on which we can iterate until the tolerance is met. Exploiting the structure of the problem is also of prime importance for reaching the goal of moderate complexity.

Given a unstable polynomial in companion matrix form A , the problem can be formulated as finding the companion matrix X such that X is stable and the squared Frobenius norm of $X - A$ is minimum.

The need for characterizing the set of stable matrices is apparent and is achieved using the well-known Lyapunov functions. Given a starting point, we built convex approximations of this stable set we defined. Convex approximations are at the core of convex optimization and are derived from the theory of barrier function. In our case, those approximations are Dikin ellipsoid obtained by computing the Hessian of a log-det barrier function. The main advantage of those approximations is that they are contained in the domain on which the barrier is defined [1]. Hence, once the approximation is built, any search direction gives an acceptable stable solution.

REFERENCES

- [1] P. Gahinet and A. Nemirovski, *The Projective Method for solving linear matrix inequalities*, Mathematical Programming (1997), 163–190.

*Joint work with Paul Van Dooren and Yurii Nesterov, ICTEAM, Université Catholique de Louvain, 4 Av Georges Lemaître, 1348 Louvain-la-Neuve, Belgium. Email adress : fx.orban@uclouvain.be

A fully-implicit Galerkin-characteristic method for nonlinear convection-dominated problems

Ahmed-Naji Ould Esouid*

Abstract

Our goal in the present work is to develop a robust numerical method to approximate solutions to the nonlinear convection-dominated problems. The key idea is to combine the modified method of characteristics and finite element discretization. In the modified method of characteristics, the time derivative and the advection term are combined as a directional derivative along the characteristics, leading to a characteristic time-stepping procedure. Consequently, the method allows for large time steps in a simulation without loss of accuracy, and eliminates the excessive numerical dispersion and grid orientation effects occurred in many upwind methods, compare for example [2, 3, 1]. A finite element characteristic method has been successfully applied for solving the unsteady semi-linear convection-dominated problems in [2]. Application of this method to shallow water flows has recently been investigated by the authors in [1]. In the current work we propose a new finite element method for the numerical simulation of nonlinear convection-dominated problems. To stabilize the method we consider the modified method of characteristics in the same manner as implemented in [2, 1]. The main drawback of these techniques is that, due to explicit time marching, the method is subject to the CFL stability conditions that put a restriction on the size of timesteps taken in numerical simulations. For problems that need long real-time computations (weeks and months) as those arising in tidal models, this restriction deteriorates the robustness and effectiveness of the method for solving such problems. Therefore, our new method differs from the approach in [2, 1] in that the time integration was based on the explicit method, whereas in the present work we use a fully-implicit time stepping scheme. The discrete system is formulated as a fixed point problem and a Newton-GMRES method is implemented for its numerical solution. We examine the performance of the proposed Galerkin-characteristic method for several test examples in nonlinear convection-dominated problems. We also apply the method to the numerical simulation of tidal flows in the Strait of Gibraltar. The governing equations are derived from the incompressible Navier-Stokes equations with assumptions of shallow water flows including bed frictions, eddy viscosity, wind shear stresses and Coriolis forces.

REFERENCES

- [1] M. El-Amrani and M. Seaïd. *An Essentially Non-Oscillatory Semi-Lagrangian Method for Tidal Flow Simulations*. International Journal for Numerical Methods in Engineering, 81 (2010) 805-834.
- [2] M. El-Amrani and M. Seaïd. *A Galerkin-characteristic method for large-eddy simulation of turbulent flow and heat transfer*. SIAM J. Sci. Comp., 30 (2008) 2734-2754.
- [3] M. El-Amrani and M. Seaïd. *Numerical simulation of natural and mixed convection flows by Galerkin-characteristic method*. Int. J. Num. Meth. Fluids, 53 (2007) 1819-1845.

*FS Tetouan, University Tetouan, Morocco.

Joint work with M. El-Amrani and N. Yebari from FST Tetouan, University, Tangier and Mohammed Seaïd from School of Engineering and Computing Sciences, University of Durham, UK).

An output controllability of bilinear systems: A numerical approach and simulations

Maawiya Ould Sidi *

Abstract

In this work, we consider a regional controllability problem for a class of distributed bilinear systems evolving in a spatial domain Ω . A feedback control is used to steer the system state close to a desired profile at a final time T , only on a subregion $\omega \subset \Omega$ which may be interior or on the boundary of the system domain. Our purpose is to prove that an optimal control exists, and characterized as a solution to an optimality system. Numerical algorithm is given and successfully illustrated by simulations.

REFERENCES

- [1] Addou, A. and Benbrik, A., *Existence and uniqueness of optimal control for a distributed-parameter bilinear system*, Journal of dynamical and control systems, (2002), pp. 141-152
- [2] Ball, J. M. Marsden, J.E. and Slemrod, M. *Controllability for Distributed Bilinear systems*, SIAM J. on control and opt., (1982), pp. 575-597
- [3] Bradly, M. E. and Lenhart, S. *Bilinear optimal control of a Kirchhoff plate to a desired profile*, Journal of Optimal control applications & methods, (1997), pp. 217-226
- [4] Buikis, A. Fitt, A.D. and Ultanova, N. *A model of oil burnout from glass fabric*, Proc. Int. Conf. of the 9th Intern. ECMI-96, Lyngby, Denmark, (1997), pp. 150-157
- [5] Lenhart, S., *Optimal control of a convective-diffusive fluid problem*, Math. Models Method Appl. Sci., (1995), pp. 225-237
- [6] Zerrik, E. Ould Sidi, M., *An output controllability of bilinear distributed system*, International Review of Automatic Control, (2010), pp. 466-473.
- [7] Zerrik, E. Ould Sidi, M. *Regional controllability of linear and semi linear hyperbolic systems*, Int. Journal of Math. Analysis, (2010), pp. 2167-2198.

*Joint work with El Hassan Zerrik. MACS Team Sciences Faculty, Moulay Ismail University, Meknes Morocco. e-mail: maawiya81@gmail.com

Solvents of non-monic matrix polynomials

Edgar Pereira *

Abstract

Let $P(X)$ be a matrix polynomial of degree m

$$P(X) = A_m X^m + \dots + A_1 X + A_0, \quad (1)$$

where A_m, \dots, A_1, A_0 are complex matrices of order n and the argument X is also a complex matrix. A very special case of this definition is to consider the argument as a complex variable λ , this $P(\lambda)$ is also known as lambda matrix. An matrix S , such that $P(S) = 0_n$ is called a solvent of $P(X)$. The characterization of solvents can be done in terms of the spectral properties of $P(\lambda)$ [1]. We develop a theory of solvents of non-monic matrix polynomials in terms of the concept of eigenpair [4]. We also study some iterative methods for the computation of such solvents ([2] and [3]).

REFERENCES

- [1] I. Gohberg, P. Lancaster and L. Rodman, *Matrix Polynomials*, Academic Press, New York, 1982.
- [2] W. Kratz and E. Stickel, *Numerical solution of matrix polynomial equations by Newton's method*, IMA Journal of Numerical Analysis 7 (1987) 355-369.
- [3] F. Marcos and E. Pereira, *A fixed point method to compute solvents of matrix polynomials*, Mathematica Bohemica 135 (2010) 355-362.
- [4] E. Pereira, *On solvents of matrix polynomials*, Applied Numerical Mathematics 42 (2003) 197-208.

*Departamento de Informática, Universidade da Beira Interior - 6201-001 Covilhã, Portugal, and Instituto de Telecomunicações - 6201-001 Covilhã, Portugal. (edgar@di.ubi.pt)

A domain decomposition method for the Canadian global numerical weather prediction model

Abdessamad Qaddouri *

Abstract

Currently, the deterministic Global Environmental Multiscale (GEM) operational model (Yeh *et al.* 2002) uses the latitude-longitude grid system which leads to singularities and convergence of meridians in the polar regions. Using today's computer architectures with distributed memory, this may even result in an unbalanced computational load in the context of domain decomposition with message passing interfaces. In this paper, we use a domain decomposition method to solve the system of primitives equations in the context of atmospheric modeling, using the Yin-Yang grid on the sphere (Kageyama *et al.* 2004). The solution of the global problem is obtained by, iteratively solving the corresponding two subproblems separately on the Yin and the Yang subgrids and updating the values at the interfaces. Because the two subgrids of the Yin-Yang grid do not match, the update is done by bicubic-Lagrange interpolation and this corresponds to the Dirichlet interface condition. To improve the performance of the classical domain decomposition methods we use optimized Schwarz methods (Qaddouri *et al.* 2008) and we replace the fixed point iterative formulation by a Krylov method, implemented by substructuring the algorithm in terms of interface unknowns (Qaddouri 2008). Preliminary results are encouraging and demonstrate that when comparing to observations, the new Yin-Yang system model performs as well as the GEM global model. Our model parallelization is performed by a hybrid use of MPI and OpenMP

REFERENCES

- [1] Kageyama A, Sato T, *The "Yin-Yang grid": An overset grid in spherical geometry* *Geochem. Geophys. Geosyst.* **5** (2004) Q09005, doi:10.1029/2004GC000734.
- [2] Qaddouri A, Laayouni L, Loisel S, Côté J, Gander M-J, *Optimized Schwarz methods with an overset grid for the shallow-water equations: Preliminary results*, *Appl. Numer. Math.*, (2008) **58** 459-471.
- [3] Qaddouri A., *Optimized Schwarz Methods with the Ying-Yang grid for Shallow Water equations*, *Domain decomposition methods in science and engineering XVII* (2008) 347-353.
- [4] Yeh K-S, Côté J, Gravel S, Méthot A, Patoine A, Roch M, Staniforth A, *The CMC-MRB global environmental multiscale (GEM) model: Part III – Nonhydrostatic formulation.*, *Mon. Wea. Rev.* (2002) 339-356.

*Meteorological Research Division, Atmospheric Science and Technology Directorate, Environment Canada, Dorval, Québec, Canada

A theory for preconditioning the inner-iteration in inexact inverse subspace iteration for generalized eigenproblems

Mickaël Robbé *

Abstract

This work focuses on the inner-iteration that arises in inexact inverse subspace iteration for computing a small deflating subspace of a large matrix pencil. The inner-iteration is illustrated by preconditioned GMRES. Sufficient conditions on the preconditioner are provided that help to maintain the number of iterations needed by GMRES to approximately constant. Further conditions lead to a small constant and hence to an efficient preconditioner. Several numerical examples of such preconditioners are given to illustrate the theory.

*Joint work with M. Sadkane, Université de Brest. Laboratoire de Mathématiques. CNRS UMR 6205. 6, Av. Le Gorgeu. CS 93837. 29238 Brest Cedex 3. France.

Multiresolution analysis and supercompact multiwavelets for surfaces over non-uniform meshes

Miguel L. Rodríguez *

Abstract

It is a well-known fact that Haar wavelet can exactly represent any piecewise constant function. Beam and Warming proved later, in [1], that the supercompact wavelets can exactly represent any piecewise polynomial function in one variable. Higher level of accuracy is attained by higher order polynomials of supercompact wavelets. The orthogonal basis used by these authors was defined as separable functions given by the product of three Legendre polynomials. In [2], the authors developed an extension of the work [1] to the case of surfaces defined over uniform meshes of the domain of the surface. Such construction keeps the same advantages attained by [1] in relation with orthogonality, short support, approximation of surfaces with no border effects, detection of discontinuities, higher degree of accuracy and compressibility. In the present work we propose to extend the multiresolution scheme developed for surfaces in [2] to the case of non-uniform meshes.

REFERENCES

- [1] R. M. Beam and R. F. Warming, *Multiresolution analysis and supercompact multiwavelets*, SIAM J. Sci. Comput. 22(4) (2000) 1238–1268.
- [2] M. A. Fortes and M. Moncayo, *Multiresolution analysis and supercompact multiwavelets for surfaces*, JCAM (2011), accepted for publication, DOI: 10.1016/j.matcom.2010.12.022.

*Joint work with M. A. Fortes, Department of Applied Mathematics, University of Granada, 18071 Granada, Spain.

Existence of solutions for a seawater intrusion problem in a free aquifer

Carole Rosier *

Abstract

In this paper we propose a model to simulate the movement of the saltwater front in coastal free aquifers.

In a first approach, we will assume that saltwater and freshwater are no miscible then the domains occupied by each fluid are separated by an interface called sharp interface. This modeling approach does not describe the nature and the behavior of the transition zone but does give information concerning the movement of the saltwater front. That is an important point for the control of seawater intrusion and for the optimal exploitation of fresh groundwater.

Let us mention that the seawater intrusion problem has been treated in the case of confined aquifers (Cf. ([2])). In this case, the confined aquifer is bounded by two horizontal and impermeable layers. The upper surface corresponds to $z = h_t$ and the lower to $z = h_b$, $h_b - h_t$ is the thickness of the aquifer assumed to be such that $(h_b - h_t) > \delta > 0$ then $T_s(h) = h_b - h$ is the thickness of saltwater zone.

For a free aquifer, the shape and position of the phreatic surface are *a priori* unknown. Since the pressure on the phreatic surface is taken as atmospheric (with $p_{atm} = 0$), the piezometric head at a point on the upper surface is equal to its elevation (Cf. ([1])).

The model is formulated in terms of a two-dimensional coupled system consisting of degenerate parabolic equations. We establish the global in time existence of variational solutions for this problem. The analysis of this system presents two difficulties: the coupling between equations and the degeneracy due to the possibility to have no saltwater in some zones of the free aquifer. We propose the Lagrange $P1$ Finite element method to numerically solve the coupled system. We establish a priori estimates in order to justify the choice of the approximation. Finally, we use the package FreeFem++ to test the efficiency and the accuracy of the model.

REFERENCES

- [1] J. Bear, Conceptual and mathematical modeling, in : J. Bear, AHD Cheng, S. Sorek, D. Ouazar, I. Herrera; *Seawater intrusion in coastal aquifers - concepts, Methods and Practices*, Kluwer academic publishers, Netherlands, 1999.
- [2] K. Najib, C. Rosier; *On the global existence for a degenerate elliptic-parabolic seawater intrusion problem*, Mathematics and computers in simulation, to appear.

*Université ULCO, Calais, France. Joint work with K. Najib, ENIM B.P.753 Rabat-Agdal, Maroc

Orthogonalization with a non-standard inner product with the application to approximate inverse preconditioning

Miro Rozložník *

Abstract

In this contribution we review the most important schemes used for orthogonalization of column vectors (stored in the matrix B) with respect to the non-standard inner product (induced by some symmetric positive definite matrix A) and give the worst-case bounds for corresponding quantities computed in finite precision arithmetic. We formulate our results on the loss of orthogonality and on the factorization error for the classical Gram-Schmidt algorithm (CGS), modified Gram-Schmidt algorithm (MGS) algorithm and for yet another variant of sequential orthogonalization, which is motivated originally by the AINV preconditioner and which uses oblique projections. Although all orthogonalization schemes are mathematically equivalent, their numerical behavior can be significantly different. It follows from our analysis that while the factorization error is quite comparable for all these schemes, the orthogonality between computed vectors can be significantly lost and it depends on the condition number $\kappa(A)$. This is the case also for the expensive implementation based on eigenvalue decomposition (EIG) and Gram-Schmidt with reorthogonalization (CGS2). The classical Gram-Schmidt algorithm and AINV orthogonalization behave very similarly and generate vectors with the orthogonality that besides $\kappa(A)$ depends also on the factor $\kappa(A^{1/2}B)\kappa(B)$ (it essentially means the quadratic dependence on the condition number of the matrix $A^{1/2}B$). Since the orthogonality in the modified Gram-Schmidt algorithm depends only linearly on $\kappa(A^{1/2}B)$, MGS appears to be a good compromise between expensive EIG or CGS2 and less accurate CGS or AINV. Indeed in the context of approximate inverse preconditioning the stabilization of AINV has led to the SAINV algorithm which uses exactly the MGS orthogonalization. We treat separately the particular case of a diagonal A which is extremely useful in the context of weighted least squares problems. One can show then that local errors arising in the computation of a non-standard inner product do not play an important role here and that the numerical behavior of these schemes is almost identical to the behavior of the orthogonalization schemes with the standard inner product. For all these results we refer to [1].

REFERENCES

- [1] J. Kopal, M. Rozložník, A. Smoktunowicz and M. Tůma. *Rounding error analysis of orthogonalization with a non-standard inner product*. 2010, submitted to Num. Math. (see also <http://www.cs.cas.cz/miro/krst10.pdf>)

*Joint work with Jiří Kopal, Miroslav Tůma and Alicja Smoktunowicz, Institute of Computer Science, Czech Academy of Sciences, Prague, Czech Republic, email: miro@cs.cas.cz.

Quadratic choreographies

Philippe Ryckelynck *

Abstract

Discrete variational problems constitute an active field of research and have been thoroughly studied by many authors. One of the basic ideas consists in replacing the derivative $\dot{\mathbf{x}}(t)$ of the dynamical variable $\mathbf{x}(t)$ with a three terms scale derivative $\square_{\varepsilon, \mathcal{Q}}\mathbf{x}(t) = \sum_{i=-N}^N c_i \mathbf{x}(t + i\varepsilon) \chi_{-i}(t)$, for some time delay ε . In this way, the principle of least action may be extended to the case of non-differentiable dynamical variables.

In this talk, we study the equations of motion occurring when we consider a fairly quadratic lagrangian \mathcal{L} of n particles in \mathbb{R}^d , where d denotes the “physical” dimension. The particles interact each to the other through quadratic terms in \mathcal{L} . We investigate the existence of pseudo-periodic solutions of classical and discrete Euler-Lagrange equations. The search for periodic solutions of those equations leads to complicated problems in linear algebra. We focus on a problem which mix solving the matrix equation $\exp(i\Omega T) = I_d$ together with generalized eigenvalues problems linking the n matrices $\exp(ij\frac{T}{n}\Omega) \in \mathbb{C}^{d \times d}$, $j = 1, \dots, n$. Numerical experiments are provided.

*Univ. Lille Nord de France, F-59000 Lille, France. ULCO, LMPA, F-62000 Calais, France. ryckelynck@mpa.univ-littoral.fr. Joint work with Laurent Smoch, Univ. Lille Nord de France, F-59000 Lille, France. ULCO, LMPA, F-62000 Calais, France. smoch@mpa.univ-littoral.fr

Töeplitz minimal eigenvalues in signal and image processing

Khalid Saeed *

Abstract

Effertz [1] explored the application of bounded series theory due to Carathéodory [2] on Brune positive real function *prf* to find the necessary and sufficient conditions for their coefficients. His solution was based on the results of Carathéodory and Töeplitz. Effertz solution is of interest in both circuit theory and image processing. The results and assertions present an appropriate mathematical model derived from these analytical functions. The author has proved that and shown their applications in digital filter design, speech signal processing, speaker identification in addition to the image processing. The relations between Töeplitz matrices and Carathéodory functions were used to prove and apply Brune function. Töeplitz matrix lowest eigenvalues are constructed by the coefficients of the bounded power series representing Carathéodory function and hence Brune function to establish a new simple and general algorithm for testing the nonnegativeness of real rational functions. The interest in these methods has recently drawn the attention of researchers due to the increasing demand in electrical and mechanical network synthesis.

Theorem 1: $f(s)$ is *prf* if it is real for real s and $Re[f(s)] \geq 0$ if $Re(s) \geq 0$.

The author has modified this theorem with an easy-to-implement algorithm. The worked out approach is based on the transformation of Brune into Carathéodory function (regular in $|p| < 1$) of Taylor's series type. The coefficients of this series form the elements of Töeplitz matrix, for which the minimal eigenvalues λ 's are calculated. Then it is shown that the sequence of λ 's decreases monotonically and that $\lim_{N \rightarrow \infty} \lambda_N = \min\{Re[f(j\omega)]\}$.

Assertion 1: For f to be *prf*, it is necessary that $\lambda_k \geq 0$ for $0 \leq k < \infty$.

Lemma 1: It is sufficient to test λ_k in $0 \leq k \leq i$ and that $\lambda_i = \lim_{n \rightarrow \infty} \lambda_n$.

Lemma 2: If $\lim_{i \rightarrow N} \lambda_i \geq 0$, then $\lambda_k \geq 0$ for $0 \leq k < i$ and hence f is *prf*.

Direct Application in image processing: The λ sequence can represent any object as its image feature vector for classification and recognition.

REFERENCES

- [1] F.H. Effertz, *On the Synthesis of Networks Containing Two Kinds of Elements*, Sympos. Modern Network Synthesis, Pol. Inst. of Brooklyn (1955), 145-173.
- [2] C. Carathéodory, *ber den Variabilitätsbereich der Koeffizienten von Potenzreihen die Gegebene Werte nicht Annehmen*, Mathematische Annalen, vol. 64 (1907), 95-115.

*AGH University of Science and Technology, Krakow, Poland

Error analysis of SRDECO Algorithm

Ahmed Salam *

Abstract

The SRDECO algorithm computes a SR decomposition based on three kind of symplectic transformations and constitutes the central task of a symplectic QR like algorithm for the solution of real algebraic Riccati equation. The purpose of this paper is to provide a detailed error analysis of the SRDECO algorithm, showing and predicating its behavior. Numerical experiments are given.

REFERENCES

- [1] P. Benner and H. Fassbender, *An implicitly restarted symplectic Lanczos method for the Hamiltonian eigenvalue problem*, Linear Algebra and its Applications Vol. 263 (1997) 75-111
- [2] A. Bunse-Gerstner and V. Mehrmann, *A symplectic QR like algorithm for the solution of the real algebraic Riccati equation*, IEEE Trans. on Automatic Control, V ol. AC-31, n12 (1986) 1104-1113.
- [3] A. Salam and E. Al-Aidarous, *Error analysis and computational aspects of SR factorization via optimal symplectic Householder transformations* , ETNA Vol. 33 (2009) 189-206.

*Univ Lille Nord de France, ULCO, LMPA, BP 699, 62228 Calais, Cedex France. Joint work with Eman Al-Aidarous, Mathematics Department, King Abdul Aziz University, P O Box 80203, Jeddah 21589, Saudi Arabia.

Identification of parameters in hydrogeology: using genetic algorithm.

Wenddabo Olivier Sawadogo *

Abstract

The problem of numerical modeling of flow and transfer of substances through a porous medium requires knowledge of the physical parameters of geological layers to assess the impact of a possible spread of pollutants. As these parameters (permeability, storage, etc..) are not directly accessible, a method of the envisaged methods consists in finding them by resolving an inverse problem.

In most cases, the boundary values and the value of the source term are known. In this work, we assume that these values in addition to permeability are unknown and we propose to determine them in 2-D using genetic algorithm. We solved the direct problem using the finite elements of Galerkin on Freefem++ and the genetic algorithm was programmed in Matlab. Then we established a communication between Matlab and Freefem++ to solve the problem of parameter identification.

We tested these programs on actual data used in the project TRANSPOL II (INERIS 2003). We obtain values more accurate than the method used in this project.

Keywords: hydrodynamics parameters, inverse problem, genetic algorithm.

REFERENCES

- [1] Adrian Dan, Patrick Goblet *Programme TRANSPOL II (INERIS 2003)*, Rapport final Mai 2004.
- [2] F. Hetch, O. Pironneau *FreeFem++*, <http://www.freefem.org>.
- [3] Latifa Oulladji Ales Janka Jean-Antoine Désidéri Alain Dervieux, *Optimisation aérodynamique par algorithmes génétiques hybrides: application à la réduction d'uncritère de bang sonique*, Rapport de recherche INRIA No 4884, Juillet 2003.
- [4] Owe Axelsson, Vincent A. Barker, *Finite element solution of boundary value problems: theory and computation*, Society for Industrial and Applied Mathematics, 2001.
- [5] E. Ledoux, *Moèles mathématiques en hydrogéologie*, Centre d'Informatique Géologique École Nationale Supérieure des Mines de Paris, 2003.

*Laboratoire de Mathématiques Appliquées et d'Informatique, BP: 549, Av. Abdelkarim Elkhatabi, Guéliz Marrakech, Maroc. Laboratoire d'Analyse Numérique, d'Informatique et de Biomathématique, Université de Ouagadougou, 03 B.P. 7021 OUAGADOUGOU 03, Burkina Faso wenddabo81@gmail.com.

Joint work with Nouredine ALAA, Laboratoire de Mathématiques Appliquées et d'Informatique, FST-Marrakech, Maroc.

Discussion on optimal control via direct search optimization

Emile Simon *

Abstract

This work considers the idea of performing optimal control design via direct search (DS) methods for Linear Time Invariant (LTI) systems. It has been suggested surprisingly recently for the Static Output Feedback (SOF) problem in [3]. There it appears that this idea is pretty successful for SOF stabilization. It has also been used for some other control problems, with generally quite satisfying results. However DS methods may fail to converge to a local optimum and stagnate instead to a 'dead point'. Such particular cases can be found i.e. in [1] where non-smoothness causes the failure of DS methods.

The motivation is to see how well DS methods compare with other techniques for optimal control design. For problems with limited number of states and that admits a Linear Matrix Inequality representation, it is fair to say that interior point methods are the best. However when the number of states increases or when the problems yield a non-convex set of solution and non-smooth cost function, other methods compete for these still open problems. Here will be illustrated how DS methods (mostly the Nelder-Mead algorithm and a bit Torczon's multidirectional search) can compete with a method considered (one of) the most numerically efficient for SOF and fixed-order problems: Hifoo [2] (an alternative would be [1] but there are no freely available implementations yet, unlike Hifoo). Around these results, the discussion will be about what improvements can be made to more often avoid non-stationary solutions or find better optima while keeping reasonable computational times. Along this discussion on the optimization methods, more control-specific concerns will be addressed.

Good results are for example obtained with large systems: SOF stabilization of unstable models with 4489 states. In general, classical SOF problems like stabilization but also \mathcal{H}_2 and \mathcal{H}_∞ norm will be shown to be solved very well. For fixed-order design, clearly the increasing number of variables increases the difficulty for the DS methods (and the number of minima). However good results are still attainable and two possibilities to try to improve the resolution these problems will be suggested.

REFERENCES

- [1] P. Apkarian and D. Noll, "Controller design via nonsmooth multidirectional search," *SIAM J. Control Optim.*, vol. 44, no. 6, pp. 1923–1949, 2006.
- [2] D. Arzelier, G. Deaconu, S. Gumussoy, and D. Henrion, "H₂ for hifoo," in *IFAC World Congress*, August 2011.
- [3] D. Henrion, "Solving static output feedback problems by direct search optimization," in *CCA, Munich*, 2006.

*Department of Applied Mathematics, Ecole Polytechnique de Louvain (UCL), 1348 Louvain-la-Neuve, Belgium, Emile.Simon@ucLouvain.be

Iterative refinement for symmetric saddle point problem

Alicja Smoktunowicz *

Abstract

We study the numerical properties of some iterative refinement techniques for improving numerical solutions of a symmetric saddle point problem of the form

$$\begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix},$$

where $A(m \times m)$ is symmetric positive definite and $B(m \times n)$ has full column rank, $n = \text{rank}(B) \leq m$.

Often existing numerical algorithms for solving saddle point problems do not take into account the symmetry and block structure of the linear system. Numerical stability aspects of direct or iterative methods are not precisely known. We give blockwise analysis of the saddle point problem, introducing blockwise condition numbers which measure the sensitivity of x and y with respect to the perturbations of the blocks A and B . We prove that under natural requirements iterative refinement techniques are able to produce blockwise stable solutions in floating point arithmetic.

Extensive numerical experiments in MATLAB will be presented to compare the performance of some methods for the saddle point problem.

REFERENCES

- [1] M. Benzi, G. H. Golub, and J. Liesen, *Numerical solution of saddle point problems*, Acta Numer. 14 (2005) 1-137.
- [2] Å. Björck and C.C. Paige, *Solution of augmented linear systems using orthogonal factorizations*, BIT 34 (1994) 1-24.
- [3] A. Smoktunowicz, *Block matrices and symmetric perturbations*, Linear Algebra Appl. 429 (2008) 2628-2635.
- [4] M. Szularz, A. Smoktunowicz and E. Pawelec, *An inverse structured perturbation problem for the linear system $A^T Ax = b$* , Demonstratio Math. XLIII (4) (2010) 755-763.

*Faculty of Mathematics and Information Science, Warsaw University of Technology, 00-661 Warsaw, Plac Politechniki 1, Poland, e-mail: smok@mini.pw.edu.pl.

Joint work with Michael West and Agata Smoktunowicz, School of Mathematics, University of Edinburgh, Edinburgh, Scotland EH9 3JZ, UK, e-mail: A.Smoktunowicz@ed.ac.uk

Parallel solution of problem with unilateral constraints

Pierre Spiteri *

Abstract

The present study deals with the solution of problem arising in fluid mechanics with unilateral constraints on the boundary. The problem to solve consists to minimize a functional in a closed convex set included in a three-dimensional domain. The characterization of the solution leads to the solution of the following time dependent variational inequality:

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} - \Delta u(x,t) = f(x,t), \text{ everywhere in } \Omega \text{ and for } 0 < t \leq T, \\ u(x,t) \geq \psi(x), \frac{\partial u(x,t)}{\partial n} \geq 0, (u(x,t) - \psi(x)) \cdot \frac{\partial u(x,t)}{\partial n} = 0, \forall x \in \Gamma_1 \text{ and } \forall t > 0, \\ u(x,t) = \psi(x), \forall x \in \Gamma_0 \text{ and } \forall t > 0, \\ u(x,0) = u_0(x), \forall x \in \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded domain and $\Gamma = \Gamma_0 \cup \Gamma_1$ is the boundary of Ω . An implicit scheme is used for the discretization of the time dependant part of the operator and the problem is then reduced to the solution of a sequence of stationary elliptic operator. For the solution of each stationary problem, an equivalent form of the minimisation problem is formulated as the solution of a multivalued equation, obtained by the perturbation of the previous elliptic operator by a diagonal monotone maximal multivalued operator. The spatial discretization of such problem by appropriate scheme leads to the solution of large scale algebraic systems. According to the size of these systems, parallel iterative asynchronous and synchronous subdomain methods are carried out on distributed architectures; in the present study subdomain methods without and with overlapping (like Schwarz alternating methods or more generally multisplitting methods) are considered. The convergence of the parallel iterative algorithms is analysed by various approaches, like contraction and partial ordering techniques. Finally the parallel experiments are presented.

REFERENCES

- [1] GIRAUD L., SPITERI P., "Résolution parallèle de problèmes aux limites non linéaires", M2 AN, vol. 25, pp. 579–606, 1991.
- [2] MIELLOU J., EL BAZ D., SPITERI P., "A new class of asynchronous iterative algorithms with order interval", Mathematics of Computation, vol. 67-221, pp. 237-255, 1998
- [3] MIELLOU J., SPITERI P., "Un critère de convergence pour des méthodes générales de point fixe" M2AN, vol. 19, pp. 645–669, 1985.

*IRIT - ENSEEIHT, UMR CNRS 5505, 2 rue Camichel, BP 7122, 31071 Toulouse (France).

Joint work with M. Chau Advanced Solutions Accelerator, 199 rue de l'Oppidum, 34170 Castlenau le Lez (France), A. Laouar Faculté des Sciences, Département de Mathématiques, Université d'Annaba, Laboratoire LANOS, BP 12, 23 000 - Annaba, Algérie and T. Garcia IRIT - ENSEEIHT, UMR CNRS 5505, 2 rue Camichel, BP 7122, 31071 Toulouse (France).

Sequential and parallel methods for 4D anisotropic diffusion of dynamic PET images

Clovis Tauber *

Abstract

In the present study, we propose accurate and fast computation of an original set of coupled 3D problems of anisotropic diffusion. Existing spatial filters used for dynamic positron emission tomography (PET) image sequences of the brain are generally based on the voxel intensity of a single timeframe, which is not necessarily a good representation of the underlying physiological process across time. 4D methods have been proposed that either filter temporally adjacent voxels or fit the intensity using models of the expected variations of intensity over time. The proposed talk deals with an original spatiotemporal robust anisotropic diffusion in which 3D spatial filtering is based on the distances between vectorial representation of the voxel intensity across time. Consequently, if m denotes the number of time frames, at each time step, it is necessary to solve m coupled non linear boundary values problems of diffusion in a parallelepipedic domain $\Omega = [0, a_1] \times [0, a_2] \times [0, a_3]$ with Neumann boundary conditions

$$(P_i) \begin{cases} \frac{\partial u_i}{\partial t} - \text{div}(c(u_1, u_2, \dots, u_m) \cdot \text{grad}(u_i)) = 0, \text{ everywhere in } \Omega, 0 < t \leq T, \\ \frac{\partial u_i(x, y, z, t)}{\partial n} \Big|_{\partial\Omega} = 0, \quad \forall t \in [0, T] \text{ (Boundaries Conditions),} \\ u_i(x, y, z, 0) = u_i^0(x, y, z), \text{ (Initial Conditions),} \end{cases}$$

where $\partial\Omega$ is the boundary of the domain Ω , $u_i = u_i(x, y, z, t)$ is the intensity, $T > 0$, is a strictly positive real number and in which the positive coefficient of diffusion $c = c(x, y, z, t, u_1, u_2, \dots, u_m)$ depends on the intensities from all timeframes u_1, u_2, \dots, u_m .

In this original study, we propose spatial discretization and time marching semi-implicit discretization scheme in which the consistency and the stability of the global scheme is verified. At each time step, it is necessary to solve m large linear algebraic systems, possibly simultaneously by a parallel way, by iterative sequential and parallel algorithms. Due to the choice of the spatial discretization step, both linear algebraic systems are very well conditioned, independently of the spatial resolution of images. More particularly, we consider the use of parallel synchronous and asynchronous iterative methods for the solution of the linear systems to solve. For both previous iterative methods, we analyse convergence. Experimental sequential and parallel simulations concerning 4D anisotropic diffusion of dynamic PET medical or synthetic images are presented.

*UMRS INSERM U930 - CNRS ERL 3106 - Université François Rabelais, Tours, (France). Joint work with Ming Chau ASA - Advanced Solutions Accelerator, Montpellier (France) and Pierre Spiteri IRT - ENSEEIHT, UMR CNRS 5505, 2 rue Camichel, BP 7122, 31071 Toulouse (France).

A New iterative technique for solving nonlinear problems

Helmi Temimi *

Abstract

We present a semi-analytical iterative method for solving nonlinear differential equations. To demonstrate the working of the method we consider some nonlinear ordinary differential equations with appropriate initial/boundary conditions. We consider an approach which tries to incorporate the various tools at a problem solvers disposal, a combination of analytical, symbolic and numerical computation. In essence our method attempts to linearize the problem and then consider an iterative approach built around analytical and numerical computations. In each of the examples we demonstrate the accuracy and convergence of the method to the solution. We demonstrate clearly that the method is accurate, fast and has a high order of convergence.

*Joint work with Ali R. Ansari, Department of Mathematics & Natural Sciences, Gulf University for Science & Technology, P.O. Box 7207, Hawally 32093, Kuwait.

Using proper orthonormal decomposition in decoupling dynamical systems

Pham Toan *

Abstract

We investigate the proper orthogonal decomposition (POD) as a useful tool in decoupling large dynamical systems suitable for parallel computing. POD is well known to be applied to model reduction for different applications. It uses snapshots of the solution at previous time steps to generate a low dimensional approximation space for the solution. Here we focus on the potential of this method in order to decouple a dynamical system into dynamical subsystems split on a multiprocessors architecture. Each processor is in charge of a subsystem in a non reduced form while it keeps a representation of the other subsystems managed by the other processors under a reduced form. The analysis of the behavior of the error on the decoupling of the dynamical system using the POD, allows us to define a mathematical criterion for updating the basis of the reduced model. Moreover, we enhance a technique to update the POD basis on the fly in order to retrieve most part of dynamics of the solution on the simulation's time interval. Some parallelism results on dynamical systems as ODE and DAE show the efficiency at wall clock of the method.

Keywords

POD, reduced-order modeling, dynamical systems, parallel computation.

*UMR5208 UCBL-CNRS Institut Camille Jordan. Joint work with Damien Tromeur-Dervout.

A MATLAB program of spectral dichotomy of regular matrix pencils

Ahmed Touhami *

Abstract

Given a regular matrix pencil $\lambda\mathbf{B} - \mathbf{A}$ and a positively oriented contour γ in the complex plane, the spectral dichotomy methods applied to $\lambda\mathbf{B} - \mathbf{A}$ and γ consist in determining whether $\lambda\mathbf{B} - \mathbf{A}$ possesses eigenvalues on or in a neighborhood of γ . When no such eigenvalues exist, these methods compute iteratively the spectral projector \mathbf{P} onto the right deflating subspace of $\lambda\mathbf{B} - \mathbf{A}$ associated to the eigenvalues inside/outside γ . The computation of the projector is accompanied by the spectral norm $\|\mathbf{H}\|$ of a Hermitian positive definite matrix \mathbf{H} called the *dichotomy condition number*, which indicates the numerical quality of the spectral projector \mathbf{P} . The smaller $\|\mathbf{H}\|$ is, the better this quality.

Based on the theory proposed in [1, 2, 3, 4], the talk discusses the implementation of a new MATLAB code, called `specdicho`, gathering the main types of spectral dichotomy methods, where γ is a circle, the imaginary axis (a straight line can also be used), a parabola or an ellipse. The MATLAB program (`specdicho`) implements the algorithm of circular spectral dichotomy denoted by DICHOC and introduced in [4], to compute \mathbf{P} and \mathbf{H} . It also incorporates extensions of DICHOC to the problem of spectral dichotomy of regular pencils with respect to the above mentioned geometries. These problems are transformed into a circular spectral dichotomy problem. In all cases, we prove the equivalence between the *quality of dichotomy* of the original problem which will be measured by the parameter $\sup_{\lambda \in \gamma} \|(\lambda\mathbf{B} - \mathbf{A})^{-1}\|$ and that of the transformed one.

keywords: invariant subspace, regular matrix pencil, spectral dichotomy, spectral projector.

REFERENCES

- [1] M. AL SAYED ALI AND M. SADKANE. On a Lyapunov type equation related to parabolic spectral dichotomy. *Electron. J. Linear Algebra* (2006), 15, 134–142.
- [2] S. K. GODUNOV AND M. SADKANE. Some new algorithms for the spectral dichotomy methods. *Linear Algebra Appl.* (2003), 358, 173–194.
- [3] A. N. MALYSHEV AND M. SADKANE. On parabolic and elliptic spectral dichotomy. *SIAM J. Matrix Anal. Appl.* (1997), 18, 2, 265–278.
- [4] M. SADKANE AND A. TOUHAMI. Modification of the spectral dichotomy algorithm of regular matrix pencils by a circle. Tech. rep., Laboratoire de Mathématiques de Brest, France, (2009).

*Université Settat, Maroc. Joint work with Miloud Sadkane, Mathematics Department, University of Brest, France.

On convergence of semi-discrete scheme for one nonlinear abstract hyperbolic equation

Mikheil Tsiklauri *

Abstract

Let us consider the Cauchy problem for the following nonlinear abstract hyperbolic equation in the Hilbert space H :

$$\frac{d^2 u(t)}{dt^2} + A^2 u(t) + a \left(\|A^{1/2} u\|^2 \right) Au(t) + M(u(t)) = f(t), \quad t \in [0, T], \quad (1)$$

$$u(0) = \varphi_0, \quad \frac{du(0)}{dt} = \varphi_1. \quad (2)$$

where A is a self-adjoint, positively defined operator with the definition domain $D(A)$, which is everywhere dense in H ; $a \left(\|A^{1/2} u\|^2 \right) = \lambda + \|A^{1/2} u\|^2$, $\lambda > 0$; nonlinear operator $M(\cdot)$ satisfies Lipschitz condition; φ_0 and φ_1 are given vectors from H ; $u(t)$ is a continuous, twice continuously differentiable, searched function with values in H and $f(t)$ is given continuous function with values in H .

Abstract analogue of Kirchhoff beam equation represents a special case of equation (1).

We are searching solution of the problem (1)-(2) by the following semi-discrete scheme:

$$\frac{u_{k+1} - 2u_k + u_{k-1}}{\tau^2} + A^2 \frac{u_{k+1} + u_{k-1}}{2} + a_k A \frac{u_{k+1} + u_{k-1}}{2} + M(u_k) = f_k, \quad (3)$$

where $a_k = a \left(\|A^{1/2} u_k\|^2 \right)$, $f_k = f(t_k)$, $k = 1, \dots, n-1$, $\tau = T/n$ ($n > 1$).

Numerical realization of scheme (3) we perform by the following algorithm:

$$\begin{aligned} u_{k+1} &= (I + \alpha_1 \tau A)^{-1} (I + \alpha_2 \tau A)^{-1} g_k - u_{k-1}, \\ g_k &= 2u_k + \tau^2 (f_k - M(u_k)), \quad u_0 = \varphi_0, \quad u_1 = \varphi_0 + \tau \varphi_1 + \tau^2 / 2 \varphi_2, \\ \varphi_2 &= f_0 - M(\varphi_0) - A^2 \varphi_0 - a_0 A \varphi_0, \end{aligned} \quad (4)$$

where α_1 and α_2 parameters satisfy the following equations:

$$\begin{aligned} \alpha_1 \alpha_2 &= 1/2, \\ \alpha_1 + \alpha_2 &= \tau a_k / 2. \end{aligned}$$

Studied stability and convergence of scheme (4). There is shown that convergence order of scheme (4) in a class of smooth solutions is $O(\tau^2)$. Using this scheme, there are carried out numerical calculations for different model problems.

*Ilia State University, Kakutsa Cholokashvili Ave 3/5 Tbilisi 0162, Georgia.

Joint work with Jemal Rogava, Ilia Vekua Institute of Applied Mathematics, 2 University Street, 0186, Tbilisi, Georgia

Evolutionary computation for optimal knots allocation in smoothing splines

Olga Valenzuela *

Abstract

In this paper, a novel methodology is presented for optimal placement and selections of knots, for approximating or fitting curves to data, using smoothing splines. It is well-known that the placement of the knots in smoothing spline approximation has an important and considerable effect on the behavior of the final approximation [1]. However, as pointed out in [2], although spline for approximation is well understood, the knot placement problem has not been dealt with adequately. In the specialized bibliography, several methodologies have been presented for selection and optimization of parameters within B-spline, using techniques based on selecting knots called *dominant points*, adaptive knots placement, by data selection process, optimal control over the knots, and recently, by using paradigms from computational intelligent, and Bayesian model for automatically determining knot placement in spline modeling. However, a common two-step knot selection strategy, frequently used in the bibliography, is an homogeneous distribution of the knots or equally spaced approach [3].

In order to optimize the placement and numbers of knots required for approximation using smoothing splines, an Evolutionary Computation Paradigms (*ECP*) based on a Multi-Objective Genetic Algorithm has been developed, with the main purpose of avoiding the large number of local minima (in terms of approximation error for different system complexity or number of knots) existing in the problem of knots placement. The accuracy, computationally efficient and robustness of the algorithm presented will be compared by different experimental result, with other approaches presented in the bibliography, showing the main advantages of the proposed methodology.

REFERENCES

- [1] G. Faring, *Curves and Surfaces for CAGD*, Morgan Kaufmann, (2002).
- [2] W. Li and S. Xu and G. Zhao and L. P. Goh, *A Heuristic Knot Placement Algorithm for B-Spline Curve Approximation*, Computer-Aided Design and Applications, vol.1 (2004), pp.727-732.
- [3] Rao, SCS; Kumar, S; Kumar, M, *Parameter-Uniform B-Spline Collocation Method for Singularly Perturbed Semilinear Reaction-Diffusion Problems*, Journal of Optimization Theory and Applications, vol.146 (2010), pp.795-809.

*Department of Applied Mathematics. University of Granada, Spain. Joint work with Miguel Pasadas

A local restart procedure for iterative projection methods for nonlinear large-scale eigenproblems

Heinrich Voss *

Abstract

We consider the nonlinear eigenvalues problem

$$T(\lambda)x = 0 \tag{1}$$

where $T(\lambda) \in \mathbb{C}^{n \times n}$ is a family of large and sparse matrices depending on a real parameter λ . Similarly to linear problems, iterative projection methods of Arnoldi [2] and of Jacobi–Davidson [1] type have turned out to be efficient. Here approximations to the wanted eigenvalues and eigenvectors are obtained from projections of the eigenproblem (1) to subspaces of small dimension which are expanded in the course of the algorithm.

A crucial point in these methods for general nonlinear eigenvalue problems when approximating more than one eigenvalue is to inhibit the method to converge to the same eigenvalue repeatedly. However, if the underlying large problem is symmetric such that its eigenvalues satisfy a minmax characterization, then this property is inherited by the projected problems, and the eigenvalues can be determined safely one after the other by safeguarded iteration.

This approach hits its limitations if a large number of eigenvalues (in particular in the interior of the spectrum) of (1) is needed, since in this case one has to project the problem under consideration onto a sequence of search spaces of growing dimensions requiring an excessive amount of storage and computing time. In this presentation we propose a new local restart technique which projects problem (1) only to search spaces of limited dimension. Our presentation is restricted to the Arnoldi method, but the local restart technique applies also to any other iterative projection method. Its efficiency is demonstrated by a large conservative gyroscopic eigenvalue problem which models the behavior of a rotating tire.

REFERENCES

- [1] T. Betcke and H. Voss. A Jacobi–Davidson–type projection method for nonlinear eigenvalue problems. *Future Generation Comput. Syst.*, 20(3):363 – 372, 2004.
- [2] H. Voss. An Arnoldi method for nonlinear eigenvalue problems. *BIT Numerical Mathematics*, 44:387 – 401, 2004.

*Hamburg University of Technology, Hamburg, Germany; e-mail: voss@tu-harburg.de.
Joint work with Marta Betcke, University College.

Implementation of scatter search to the scheduling problems

Hanna Zini *

Abstract

In many considerable disciplines of the Combinatorial optimization, the evolutionary approach "Scatter Search" Proposed at the beginning by "Fred Glover" in the years 70, is proved effective for the resolution of a Variety of NP-Hard problems. We identify the scatter search Method, which bases itself on a generation Of Reference set and we implement this Approach on a scheduling problem.

Keywords: Scatter Search, heuristics, Combinatorial optimization, scheduling problem.

*Joint work with Souad El Bernoussi, Département de Mathématiques-Faculté des Sciences-Rabat

List of Participants

AAJAJ	Tarik	Morocco
ABDOUS	Belkacem	Canada
ABE	Kuniyoshi	Japan
ABSIL	Pierre-Antoine	Belgium
AGESTINO		Italy
AIT KHOUYA	Youssef	Morocco
AIT OUSSOUS	Mohammed	Morocco
AL AIDAROUS	Eman	Saudi Arabia
ALAA	Noureddine	Morocco
ALAWNEH	Ameen	Qatar
AYAD	Habib	Morocco
ALHURI	Yaser	Morocco
BASSOUR	Mustapha	Morocco
BELHAJ	Skander	Tunisia
BELHAJ	Zakaria	Morocco
BELLALIJ	Mohammed	France
BENTALEB	Youssef	Morocco
BENTBIB	Abdesslam	Morocco
BENYACOUB	Badreddine	Morocco
BORCKMANS	Pieree	Belgium
BOUHAMIDI	Abderrahman	France
BROWET	Arnaud	Belgium
CHAHLAOUI	Younes	United Kingdom
COX	Eric	USA
DE VLIAGER	Jeroen	Belgium
DEL CORSO	Gianna M.	Italy
DELMAIRE	Gilles	France
DETOURNAY	Sylvie	France
DI NAPOLI	Edoardo	Germany
DIKHAMINJIA	Nana	Georgia
DOURI	Sidi Mohamed	Morocco
EL AASSRI	Abdelwahab	Morocco
EL ALLALI	Zakaria	Morocco
EL AYADI	Rachid	Morocco
EL BERNOUSSI	Souad	Morocco
EL FATINI	Mohamed	France
EL HAJJI	Said	Morocco
EL WASSIFI	Karima	Morocco
ESTATICO	Claudio	Italy
FARINA	Raffaele	Italy
NOURI	Fatime Zohra	Algeria
FERCOQ	Olivier	France
FORTES	Miguel Ángel	Spain
GANDER	Martin	Switzerland
GULLIKSSON	Marten	Sweden
HEYOUNI	Mohammed	Morocco
IBANEZ PEREZ	María José	Espagne

IKUNO	Soichiro	Japon
IZEM	Nouh	Morocco
IZHAR	Ahmad	Saudi Arabia
JBILOU	Khalide	France
JIRANEK	Pavel	France
KANBER	Ahmed	Morocco
KOUIBIA	Abdelouahed	Spain
KUMAR	Pawan	France
LAARAJ	Mohammed	Morocco
LAAYOUNI	Lahcen	Morocco
LAKHBAB	Halima	Morocco
LARGO	Bruno	
LEFRAICH	Hamid	Morocco
LINDGREN	Carina	
LINEL	Patrice	Belgium
LOOSLI	Cédric	France
MAGLIANI	Agostino	Italy
MASHHOUR	Bani Ata	Kuwait
MASTRONARDI	Nicola	Italy
MEHRMANN	Volker	Germany
MENCHI	Ornella	Italy
MESSAOUDI	Abderrahim	Morocco
MEURANT	Gérard	France
MORIKUNI	Keiichi	Japan
MOUIDA	Abdelhaq	Morocco
NACHAOUI	MOURAD	France
NEYTCHEVA	Maya	Sweden
NOSCHESE	Silvia	Italy
NOUISSE	Otheman	Morocco
ORBAN DE XIVRY	Francois-Xavier	Belgium
OULD ESQUID	Ahmed-Naji	Morocco
OULD SIDI	Maawiya	Morocco
PASADAS	Miguel	Spain
PELEN	Rhumi	Turqey
PELEN	Neslihan	Turqey
PEREIRA	Edgar	Portugal
PHAM DUC	Toan	France
PREVOST	Marc	France
QADDOURI	Abdessamad	Canada
RAGHAY	Said	Morocco
REICHEL	Lothar	USA
ROBBE	Mickaël	France
RODRIGUEZ	Miguel L.	Spain
ROSIER	Carole	France
ROUSSEL	Gilles	France
ROZLOZNIK	Miro	Czech Republic
RYCKELYNCK	Philippe	France
SAAD	Yousef	USA
SADAKA	Rachid	Morocco
SADOK	Hassane	France
SAEED	Khalid	Poland
SALAM	Ahmed	France

SIMON	Emile	Belgium
SMOCH	Laurent	France
SMOKTUNOWICZ	Alicja	Poland
SMOKTUNOWICZ	Agata	United Kingdom
SPITERI	Pierre	France
TAUBER	Clovis	France
TEMIMI	Helmi	Koweit
TOUHAMI	Ahmed	Morocco
TSIKLAURI	Mikheil	Georgia
VALENZUELLA	Olga	Spain
VAN DOOREN	Paul	Belgium
VOSS	Heinrich	Germany
ZLEC	Alicia	Poland
ZINI	Hanna	Morocco

