The ϵ-strategy in variational analysis

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Abstract. In mathematics, it happens one cannot tackle a problem directly, for several reasons: because the original problem has no solution, because the definition of the solution itself is ambiguous, because the posed problem may have several solutions while the underlying physical or mechanical problem clearly indicates only one, etc. Therefore, an often used strategy is to circumvent the intrinsic difficulty of the problem by *perturbing it by* $\epsilon > 0$. The perturbed problem often turns out to be easier to solve or just more meanginful than the original one. Then, since ϵ is small, as always in mathematics, it remains to see what happens when $\epsilon \to 0$ or how to filter the ϵ-mathematical objects associated with the ϵ-perturbed problems. In doing so, one expects to get something of interest for the original problem, which typically corresponds to $\epsilon = 0$.

In this work, we concentrate our interest on variational (or optimization) problems which do not have solutions necessarily, but which do have (exact) approximate solutions (or solutions within $\epsilon > 0$). The question we address is: what to do with such ϵ-solutions ? We shall see how to recover all the minimizers of the relaxed version of an abstract variational problem in terms of ϵ-minimizers of the original variational problem (especially when this original solution has no solution). Applications to two approximation problems in a Hilbert space setting will be shown.

This results from joint efforts with M. LOPEZ (Universidad de Alicante, Spain) and M. VOLLE (Université d'Avignon, France). A paper will be available if some participants are interested in the detailed presentation.