Multigrid modified method of characteristics for solving advection-dominated transport problems

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Abstract

We present a new numerical method for solving unsteady advection-dominated transport problems. The method consists of an Eulerian-Lagrangian splitting of the equations along the characteristic curves. The Lagrangian stage of the splitting is treated by a modified method of characteristics, while a finite element method is used for the Eulerian stage. The combined two stages turn to a robust algorithm for accurately solving the equations and also overcome many difficulties in other conventional numerical methods to treat the advective part of the transport equations.

In our contribution, we present a new multigrid modified method of characteristics algorithm. Thus using the same mesh hierarchy for both Eulerian and Lagrangian procedures, the linear system arising from the discretization of the transport equations is solved by multilevel method using the Atkinson-Brakhage approximate inverse as a preconditioner. On the other hand a multilevel solver, is used for the interpolation at the departure points in the lagrangian stage of the algorithm. At each time step, linear systems are solved only on the coarse mesh. To be more precise our talk deals with the two following new features of modified method of characteristics:

1. Development of a multigrid modified method of characteristics solver suitable for numerical solution of advection-dominated transport problems.
2. Numerical investigation (robustness, efficiency and convergence rates) of the developed method for solving the incompressible Navier-Stokes equations and the shallow water problems.

Other Related issues like calculation of departure points, mass conservation and computer implementation will also be discussed. It should be stressed that most of the results reported here are the fruits of recent or ongoing research [1, 2, 3, 4].

References

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