Numerical Solution of Linear and Nonlinear Matrix Equations Arising in Stochastic and Bilinear Control Theory

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Abstract

The reachability and observability Gramians of stable linear time-invariant systems are well-known to be the solutions of Lyapunov matrix equations. Considering the classes of linear stochastic or bilinear control systems, studying the concepts of reachability and observability again leads to the solutions of generalized Lyapunov equations, which we will call *Lyapunov-plus-positive equations*, since the generalization consists in adding a positive operator to the Lyapunov operator in the left-hand side of these equations. Model reduction methods analogous to balanced truncation for LTI systems can be based on solving these Lyapunov-plus-positive equations. Due to the large-scale nature of these equations in the context of model order reduction, we study possible low rank solution methods for them.

We show that under certain assumptions one can expect a strong singular value decay in the solution matrix allowing for low rank approximations. We further provide some reasonable extensions of some of the most frequently used linear low rank solution techniques such as the alternating directions implicit (ADI) iteration and the extended Krlyov subspace method. These methods are compared to, or even serve as preconditioners for, tensor versions of standard Krylov subspace solvers for linear systems of equations that can also be applied efficiently in this context. By means of some standard numerical examples used in the area of bilinear and stochastic model order reduction, we will show the efficiency of the new methods. These results are mostly contained in [1].

Stochastic optimal control problems and generalizations of balanced truncation for stochastic and bilinear systems using, e.g., LQG balancing, lead to the need of numerically solving nonlinear matrix equations, where the linear part has exactly the form of a Lyapunov-plus-positive equation, while the quadratic term is as in the standard LTI case. We will briefly discuss variants of Newton's method employing any of the solvers for Lyapunov-plus-positive equations in the Newton step.

References

[1] P. Benner and T. Breiten. Low rank methods for a class of generalized Lyapunov equations and related issues. *Numerische Mathematik*, to appear. *See also:* MPI Magdeburg Preprint MPIMD/12-03, Februar 2012, http://www.mpi-magdeburg.mpg.de/preprints/abstract.php?nr=12-03&year=2012.