Hole filling on surfaces by discrete variational splines

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Abstract

The problem of adequately filling holes or completing a 3D surface may arise in all sorts computational graphics areas, like CAGD, CAD-CAM, Earth Sciences, computer vision in robotics, image reconstruction, etc.

On the other hand, the variational methods for the approximation of curves and surfaces have received considerable attention in the past few years, due to their efficiency and usefulness.

These methods are based in the minimization of a quadratic functional which usually contains mainly two terms: one of them indicates how well the curve or surface approximates a given data set and the other one controls the degree of smoothness or fairness of the curve or surface.

A wide range of minimization functionals has been proposed, derived from physical considerations (e. g. stretch energy or blending energy) or geometric entities (e. g. curve length, surface area or curvature).

Discrete smoothing D^m-splines and discrete smoothing variational splines provide specific examples of variational curves and surfaces.

These splines minimize, in a finite element space, some quadratic functionals that contain terms associated with Sobolev seminorms.

In all cases, the obtained spline functions approximate a Lagrangian data set (that is, a set of values of a known or unknown function) or a Hermite data set (that is, a set of values of a known or unknown function as well as the values of some of its partial derivatives).

Several works related to the field of filling holes have been published in the last few years.

In this work we present two different methods to fill one or more holes $H = H_1 \cup \cdots \cup H_n$ in an explicit $3D$-surface defined over $\overline{D} - H$, with $D \subset \mathbb{R}^2$ a polygonal domain, by a smooth function f.

We do the filling in two different ways: discontinuous and regular.

In the discontinuous case, for any hole H_i, we consider a local polygon $\widetilde{H}_i$ that contains H_i and such that it has not common intersection with the remainder holes.

Then, for each polygon $\widetilde{H}_i$, we consider a partition into rectangles or triangles and we obtain a discrete smoothing variational spline $\tilde{\sigma}_i$ defined on $\widetilde{H}_i$ by the minimization of a functional on a finite element space constructed from the considered partition and approximating a given point data set in $\widetilde{H}_i - H_i$.

The reconstruction will be done just putting together the original function f ouside H with each spline $\tilde{\sigma}_i$ inside H_i, for $i = 1, \ldots, n$. Obviously, the solution is not continuous but we obtain a good visual reconstruction.

In the regular case we consider a partition into rectangles or triangles of $\overline{D}$ and we obtain a discrete smoothing variational spline σ defined on D by the minimization of a functional on a finite element space constructed from this partition and approximating a given point data set in $\overline{D} - H$.

The reconstruction will be done by σ. Obviously this solution has the regularity of the finite element space and we prove the validity of this method with some numerical and graphical examples.