New Implementation of the Block GMRES method by using recursive computation of the pseudo-inverse

A. Messaoudi*

Abstract

We consider systems of linear equations with multiple right-hand sides

$$AX = B, \quad A \in \mathbb{C}^{N\times N}, \quad B \in \mathbb{C}^{N\times s}, \quad X \in \mathbb{C}^{N\times s}, \quad 1 \leq s \ll N, \tag{1}$$

which is equivalent to the s-systems of linear equations

$$Ax^{(i)} = b^{(i)}, \quad \text{for} \quad i = 1, \ldots, s. \tag{2}$$

For solving (1) several block methods have been developed: the Standard GMRES, the Global GMRES and the Block GMRES methods. We will be interested by the block GMRES method (BGMRES). We consider the matrix Krylov subspace $\mathbb{K}_k(A, R_0)$ defined as

$$\mathbb{K}_k(A, R_0) = blockspan\{R_0, AR_0, \ldots, A^{k-1}R_0\} \tag{3}$$

$$= \left\{ \sum_{i=1}^{k} A^k R_0 \gamma_i / \gamma_i \in \mathbb{C}^{s\times s} \right\}. \tag{4}$$

Then the BGMRES method is defined by

$$X_k = X_0 + Z_k,$$

where Z_k is the solution of the following least squares problem

$$\| R_0 - AZ_k \|_F = \min_{Z\in\mathbb{K}_k(A,R_0)} \| R_0 - AZ \|_F \ . \tag{5}$$

This problem (5) can be solved by using the Givens rotations. Our idea is to use the pseudo-inverse for solving this problem.

*Ecole Normale Supérieure de Rabat, Maroc.