Probabilistic metric spaces as enriched categories

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Lawvere's ground-breaking paper [2] presenting generalised metric spaces as enriched categories has motivated much work on the reconciliation of order, metric and category theory, we refer in particular to the work of Flagg *et al.* on continuity spaces and the work of the Amsterdam research group at CWI. One amazing insight of [2] is a characterisation of the notion of Cauchy completeness for metric spaces using adjoint distributors, giving further evidence to MacLane's motto "adjoints occur almost everywhere". Later on, this result was further generalised to categories enriched in a *value quantale* (see [1]): for such categories, Cauchy completeness can be equivalently described via distributors and via Cauchy nets. Using the conceptual power of adjunction, in this talk we show that many results linking adjoint distributors and Cauchy sequences (resp. nets) are valid under milder assumptions. We interpret our results in probabilistic metric spaces [3] seen as categories enriched in the quantale Δ of distribution functions, and show that in many cases categorical and classical notions coincide.

This is joint work with my PhD student Carla Reis.

References

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[2] F. W. Lawvere, *Metric spaces, generalized logic, and closed categories*, Rendiconti del Seminario Matematico e Fisico di Milano, 43 (1973), pp. 135–166, http://www.tac.mta.ca/tac/reprints/articles/1/tr1.pdf. Republished in: Reprints in Theory and Applications of Categories, No. 1 (2002) pp 1–37.

[3] B. Schweizer and A. Sklar, *Probabilistic metric spaces*, North-Holland Series in Probability and Applied Mathematics, North-Holland Publishing Co., New York, 1983.