New algorithm for computing the interpolation polynomials

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Abstract

The general Hermite (also called the Hermite-Birkoff) polynomial interpolation problem is defined as follows: n is a non negative integer, let x_i, $i = 0, 1, \cdots, n$, be $n+1$ distinct real numbers and $y_{i,k}$, for $i = 0, 1, \cdots, n$ and $k = 0, 1, \cdots, n_i$, be given real numbers. The Hermite interpolation problem for this data consists of determining a polynomial $p_{N-1} \in \mathcal{P}_{N-1}$, where $N = \sum_{i=0}^{n}(n_i+1)$, which satisfies the following interpolation conditions

$$p_{N-1}^{(k)}(x_i) = y_{i,k}, \quad i = 0, 1, \cdots, n, \quad k = 0, 1, \cdots, n_i.$$

$p_{N-1}(x)$ is given by

$$p_{N-1}(x) = \sum_{i=0}^{n} \sum_{k=0}^{n_i} y_{i,k} L_{i,k}(x),$$

The generalized Lagrange polynomials $L_{i,k} \in \mathcal{P}_{N-1}$ are defined as follows: starting with auxiliary polynomials

$$l_{i,k} = \frac{(x-x_i)^k}{k!} \prod_{\substack{j=0\\ j\neq i}}^{n} \left(\frac{x-x_j}{x_i-x_j}\right)^{n_j+1}, \; i = 0, \cdots, n, \; k = 0, \cdots, n_i,$$

put $L_{i,n_i}(x) = l_{i,n_i}(x)$, $i = 0, 1, \cdots, n$, and recursively for $k = n_i - 1, n_i - 2, \cdots, 0$

$$L_{i,k}(x) = l_{i,k}(x) - \sum_{m=k+1}^{n_i} l_{i,k}^{(m)}(x_i) L_{i,m}(x).$$

We will consider the particular case where $n_i = \mu$, for $i = 0, 1, \cdots, n$. We will give another formulation of the general Hermite polynomial interpolation problem. We will show how to solve this problem, and give a new algorithm for computing the interpolation polynomial $p_{N-1}(x)$.

This new algorithm will be called the Matrix Recursive Polynomial Interpolation Algorithm (MRPIA). Some of its properties will be studied. A method will be proposed for the case where n_i are different. Some examples will also be given.

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