A great open air model of a then projected dam lake was used to investigate the wave development in 1967 [2]. The dimensions of the model lake were in the order of tens of meters. Wave elevations only were measured by water resistance probes. Unfortunately the methods of landslide simulation and wave elevation measurement used at that time led to a rather great scatter of results. The research team and the consultant firm contented themselves by the statement that no dam overspill can occur at the then assumed initial water level and landslide size. Both have changed since. The two-dimensional numerical model was applied to solve the same situation. The results indicated that the wave elevation at dam is nearly proportional to the landslide immersed volume the second important parameter being the initial water level in the lake. Landslide thickness, shape and velocity had a negligible effect on the wave run-up on dam because the landslide was in a side valley and these landslide parameters affected only the formation of lateral waves which did not propagate in the dam direction. Using functions found by the numerical model to reduce the physical model results to a single initial water level (the water levels have been fortunately registered and preserved) the physical modelling results lost an overwhelming part of their scatter. A regression to a linear function of the landslide's immersed volume was then applied on the physical modelling results. The resulting straight line fits very well with the numerical modelling results as shown in Fig.2 (b). Confidence in both modelling methods was thus strenghtened and old experiment analysis upgraded after thirty years. The mathematical model only proved to be much cheaper and more versatile. It has been used to solve the problem in contemporary conditions.

4 Conclusion

The described numerical model is an engineering approximation to the complicated landslide caused wave development on lakes. The simulated dissipation of wave energy related to waveheight to wavelength ratio and to the instantaneous and local water depth enabled a block implementation of simple equations. Stable solutions using a fixed staggered Cartesian grid and moving domain boundaries can be made.

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Numerical Tests of an Artificial Boundary Condition for Exterior Stokes Flows

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Abstract. We present a finite element method for approximating an exterior Stokes flow. The method is based on the use of a local artifial boundary condition on a truncating sphere.

1. Theoretical Background.

Consider the Stokes system

$$-\Delta u + \nabla \pi = f, \qquad \operatorname{div} u = 0 \tag{1.1}$$

in an exterior domain $\mathbb{R}^3 \setminus \overline{\Omega}$, where Ω is a bounded polyhedron in $\mathbb{R}^3$. The case $\Omega = \emptyset$ is also admitted. We impose a Dirichlet boundary condition on $\partial\Omega$,

$$u \,|\, \partial\Omega = 0, \tag{1.2}$$

and a decay condition near infinity,

$$u(x) \to 0 \quad (|x| \to \infty). \tag{1.3}$$

Of course, the Dirichlet condition (1.2) is to be dropped if Ω is empty. In the following, we shall assume that the exterior force f is a L^2-function: $f \in L^2(\mathbb{R}^3 \setminus \overline{\Omega})^3$. In addition, we prescribe a decay condition near infinity:

$$f(x) = O(|x|^{-\epsilon-4}) \quad (|x| \to \infty) \quad \text{for some } \epsilon \in [0, \infty). \tag{1.4}$$

Then boundary value problem (1.1) - (1.3) has a unique solution in suitable Sobolev spaces:

Theorem 1.1 *(see [1, Theorem 3.3], [3, p. 271, Theorem 4.1]): There is a uniquely determined pair of functions* $(u, \pi) \in H^2_{loc}(\mathbb{R}^3 \setminus \overline{\Omega})^3 \times H^1_{loc}(\mathbb{R}^3 \setminus \overline{\Omega})$ *which satisfies the relations*

$$u \in L^6(\mathbb{R}^3 \setminus \overline{\Omega})^3, \qquad \nabla u \in L^2(\mathbb{R}^3 \setminus \overline{\Omega})^9, \qquad \pi \in L^2(\mathbb{R}^3 \setminus \overline{\Omega})$$

and solves boundary value problem (1.1) - (1.3).

This solution (u, π) of problem (1.1) - (1.3) will be called "exterior flow". Here we shall discuss the question of how to approximate this exterior flow by finite element methods.

As the first step for obtaining such an approximation, we cut off the exterior domain by a large ball B_R with radius R and center in the origin. Denote by Ω_R that part of the exterior domain which lies inside this ball: $\Omega_R := B_R \backslash \overline{\Omega}$. Then we would like to compute the restriction of the exterior flow (u, π) to this truncated exterior domain Ω_R. However, a direct computation is impossible since we do not know the boundary value of the velocity u on the artificial boundary ∂B_R. In order to overcome this obstacle, we have to impose an artificial boundary condition on the sphere ∂B_R. One may either prescribe a local boundary condition or – if the exterior force f has compact support – a non-local condition.

If f has compact support and a non-local condition is chosen, a numerical approximation of the exterior flow is usually obtained by coupling a finite element method on Ω_R with a boundary element method on ∂B_R ([13], [14], [8]). An advantage of this approach is that the radius R of the truncating sphere ∂B_R may be kept fixed. On the negative side, the coupling of a finite element and a boundary element method complicates the discretization and solution process.

Here we shall consider a local artificial boundary condition on ∂B_R. This leads to the following problem: the Stokes system

$$-\Delta v_R \,+\, \nabla \varrho_R \,=\, f \,|\, \Omega_R, \qquad \operatorname{div} v_R \,=\, 0 \tag{1.5}$$

on the truncated exterior domain Ω_R, with a Dirichlet boundary condition on $\partial\Omega$:

$$v_R \,|\, \partial\Omega \,=\, 0, \tag{1.6}$$

and with a local artificial boundary condition on ∂B_R :

$$L_R(v_R,\, \varrho_R)(x) \,=\, 0 \qquad \text{for } x \in \partial B_R. \tag{1.7}$$

Here L_R denotes a suitable boundary operator acting on v_R and ϱ_R. A first condition on the choice of L_R is that it should be possible to write problem (1.5) – (1.7) in mixed variational form so that a finite element method is sufficient to discretize this problem. The exterior force f need not have compact support in this case; it is enough to require a decay condition near infinity as stated in (1.4). On the other hand, a drawback of such an approach is that a solution $(v_R,\, \varrho_R)$ of problem (1.5) - (1.7) will not, in general, coincide with the restriction of the exterior flow to Ω_R. However, and this is the crucial point, a solution of problem (1.5) - (1.7) will be close to the exterior flow provided the artificial boundary condition (1.7) is chosen in the right way. It turned out that a good choice looks like this:

$$\begin{aligned} L_R(v, \varrho)(x) \;&:=\; 3/(2\cdot R)\cdot v(x) \\ &+\; \Big(\sum_{k=1}^{3} \big(D_k v_j \,-\, (1/2)\cdot D_j v_k \,-\, \delta_{jk}\cdot \varrho \big)(x) \cdot (x_k/R) \Big) \end{aligned}$$

for $x \in \partial B_R$. With this operator L_R, problem (1.5) - (1.7) may be written as a mixed variational problem which has a unique solution:

Theorem 1.2 *(see [1, Theorem 4.2, Lemma 4.2]): Let $R \in (0, \infty)$ with $\overline{\Omega} \subset B_R$. Define*

$$\begin{aligned} W(\Omega_R) \;&:=\; \big\{\, u \in H^1(\Omega_R)^3 \;:\; u \,|\, \partial\Omega = 0 \,\big\}, \\ a_R(v, w) \;&:=\; \int_{\partial B_R} 3/(2\cdot R)\cdot v \cdot w \;\, do_x \\ &+\; \int_{\Omega_R} \sum_{j,\,k\,=\,1}^{3} \big(D_j v_k \cdot D_j w_k \;-\; (1/2)\cdot D_j v_k \cdot D_k w_j \,\big)\, dx, \\ b_R(v, \sigma) \;&:=\; \int_{\Omega_R} -\, div v \cdot \sigma \; dx \end{aligned}$$

for $v,\, w \in W(\Omega_R)$, $\sigma \in L^2(\Omega_R)$. Then there is a uniquely determined pair of functions $(v_R,\, \varrho_R) \in W(\Omega_R) \times L^2(\Omega_R)$ such that

$$\begin{aligned} a_R(v_R, w) \,+\, b_R(w, \varrho_R) \,&=\, \int_{\Omega_R} f \cdot w \; dx \qquad \text{for } w \in W(\Omega_R), \\ b_R(v_R, \sigma) \,&=\, 0 \qquad \text{for } \sigma \in L^2(\Omega_R). \end{aligned}$$

If $v_R \in H^2(\Omega_R)^3$, $\varrho_R \in H^1(\Omega_R)$, then the pair $(v_R,\, \varrho_R)$ satisfies (1.5) - (1.7) pointwise.

The truncation error, that is, the difference between this weak solution (v_R, ϱ_R) of (1.5) - (1.7) and the exterior flow (u, π), may be estimated in the following way:

Theorem 1.3 *(see [1, Theorem 5.1]): Fix a number $S \in (0, \infty)$ with $\overline{\Omega} \subset B_S$. Then there is a constant $C = C(\Omega, S)$ such that it holds for $R \in [S, \infty)$:*

$$\big\| \, v_R \,|\, \Omega_S \,-\, u \,|\, \Omega_S \, \big\|_2 \,\le\, C \cdot R^{-3/2} \cdot \mathcal{M}(\epsilon, R),$$

with $\mathcal{M}(\epsilon, R) := 1$ if $\epsilon > 0$, and $\mathcal{M}(\epsilon, R) := 1 + |\ln R|$ if $\epsilon = 0$, where ϵ was introduced in (1.4).

Such an estimate controls the error in a vicinity of Ω and is called a "local" estimate of the truncation error. The decay rate of the truncation error given by Theorem 1.3 is best possible in the sense that it is the same rate as obtained by Goldstein [6], [7] for solutions of the Poisson equation in exterior domains. Results similar to Theorem 1.3 were proved by Nazarov, Specovius [12]. As examples for references dealing with artificial boundary conditions for the Navier-Stokes equations – albeit without estimate for the truncation error –, we cite Kračmar, Neustupa [12] and Heywood, Rannacher, Turek [9].

Once problem (1.5) – (1.7) is written in mixed variational form, it is accessible to mixed finite element methods. In order to be able to apply such a method, we

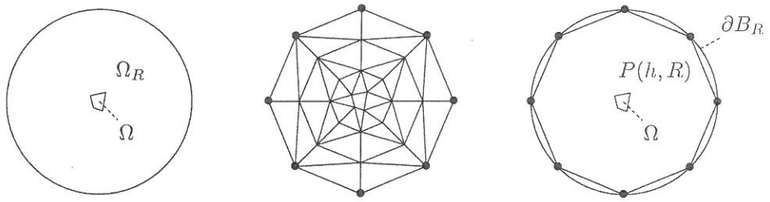

Figure 1: *Example for domains Ω and Ω_R and for a triangulation $\mathcal{T}(h,R)$ of Ω_R, with the polyhedron $P(h,R)$ related to $\mathcal{T}(h,R)$ depicted on the right (2D representation). The large dots correspond to nodes located on ∂B_R.*

introduce a family $\mathcal{T}(h,R)$ of triangulations, with $h \in (0,S)$ and $R \in \{\, 2^J \cdot S \;:\; J \in \mathbb{N},\ J > 2 \,\}$. This choice of R means that we only consider truncating spheres of radius $R = 2^J \cdot S$, with $J \in \mathbb{N},\ J > 2$. Each grid $\mathcal{T}(h,R)$ is supposed to triangulate a polyhedron $P(h,R)$ which exhausts the domain Ω_R up to some remainder set near the sphere ∂B_R; see Fig. 1 for a 2D representation of the situation we have in mind. The grid $\mathcal{T}(h,R)$ is made up of tetrahedrons $K_1(h,R)$, ..., $K_{\tau(h,R)}(h,R)$, with some number $\tau(h,R) \in \mathbb{N}$. If a tetrahedron is located near Ω, it is supposed to have a diameter of order h. Following an idea by Goldstein [6], we increase the element mesh size with increasing distance from Ω. More specifically, we suppose the following:

1.) There is a constant $\gamma \in (0,1)$ such that

$$\sup\{\, r \in (0,\infty) \;:\; B_r(x) \subset K_l \ \text{ for some } x \in \mathbb{R}^3 \,\} \;\geq\; \gamma \cdot 2^{j-2} \cdot h \qquad (1.8)$$

for $h \in (0,S)$, $J \in \mathbb{N}$ with $J > 2$, $j \in \{2, ..., J\}$, $l \in \{\, 1, ..., \tau \,\}$ with

$$K_l \cap \big(B_{2^j \cdot S} \backslash \overline{B_{2^{j-1} \cdot S}} \big) \neq \emptyset \quad \text{if } j \geq 3, \qquad K_l \cap \big(B_{2 \cdot S} \backslash \overline{\Omega} \big) \neq \emptyset \quad \text{if } j = 2,$$

where $K_l := K_l(h,\, 2^J \cdot S)$, $\tau := \tau(h,\, 2^J \cdot S)$, and $B_r(x)$ denotes a ball with radius r and center $x \in \mathbb{R}^3$.

2.) $\operatorname{diam} K_l \;\leq\; 2^{j-2} \cdot h$ for h, J, j, l, K_l as in 1.).

The preceding conditions 1.), 2.) mean that any tetrahedron $K_l(h,\, 2^J \cdot S)$ intersecting the angular region $B_{2^j \cdot S} \backslash \overline{B_{2^{j-1} \cdot S}}$ has a diameter of order $2^{j-2} \cdot h$. In particular, the element mesh size doubles from one such angular region to the next in outward direction.

Some additional technical assumptions must be imposed on our grids. For example, each vertex of $P(h,R)$ should either be a vertex of Ω or be located on

∂B_R. For further details we refer to [2]. Here we only mention that the number of gridpoints arising with Goldstein's mesh grading process is of order $h^{-3} \cdot \ln R$ ([1, Lemma6.3]). If a fixed bounded domain is considered, this factor $\ln R$ would not arise, so this factor is the price we have to pay for dealing with an exterior problem. We cannot further reduce the number of gridpoints without losing the optimal error estimates we shall state in Theorem 1.5 below.

We introduce a pair of finite element spaces $\big(W_{h,R},\, M_{h,R} \big)$ in the following way, for $h \in (0,S)$, $R = 2^J \cdot S$ with $J \in \mathbb{N},\ J > 2$:

$$\begin{aligned} W_{h,R} \;&:=\; \big\{\, v \in C^0\big(\overline{P(h,R)}\big)^3 \;:\; v \,|\, K_l \in \operatorname{span}\big(\mathcal{P}_1(K_l)^3 \cup \{ b_{K_l} \}^3 \big) \\ &\qquad\qquad \text{for } \ 1 \leq l \leq \tau(h,R) \,\big\}, \\ M_{h,R} \;&:=\; \big\{\, \varrho \in C^0\big(\overline{P(h,R)}\big) \;:\; \varrho \,|\, K_l \in \mathcal{P}_1(K_l) \ \text{ for } \ 1 \leq l \leq \tau(h,R) \,\big\}. \end{aligned}$$

Here we wrote K_l instead of $K_l(h,R)$, and we denoted by $\mathcal{P}_1(K_l)$ the space of polynomials over K_l of order less than or equal to 1. Moreover, we wrote b_{K_l} for the usual bubble function over K_l. This means that b_{K_l} is a polynomial of order 4 which vanishes on ∂K_l. Thus, over each tetrahedron K_l, our finite element spaces reduce to the Mini element.

We define the bilinear forms $a_{h,R} : W_{h,R} \times W_{h,R} \mapsto \mathbb{R}$ and $b_{h,R} : W_{h,R} \times M_{h,R} \mapsto \mathbb{R}$ in the same way as a_R and b_R, respectively (Theorem 1.2), but with the domains of integration $\partial\Omega_R$ and Ω_R replaced by $\partial P(h,R) \backslash \partial\Omega$ and $P(h,R)$, respectively.

The following theorem is a consequence of the general theory of mixed finite element methods, as presented in [4], for example.

Theorem 1.4 *(compare [1, Theorem 6.1]): Let $h \in (0,S)$ and $R = 2^J \cdot S$ for some $J \in \mathbb{N}$ with $J > 2$. Then there is a uniquely determined pair of functions $(v, \varrho) = \big(v(h,R),\, \varrho(h,R) \big)$ such that $v \in W_{h,R},\ \varrho \in M_{h,R}$ and*

$$\begin{aligned} a_{h,R}(v,w) \;+\; b_{h,R}(w,\varrho) \;&=\; \int_{P(h,R)} f \cdot w \; dx \qquad \text{for } \ w \in W_{h,R}, \\ b_{h,R}(v,\sigma) \;&=\; 0 \qquad\quad \text{for } \ \sigma \in M_{h,R}. \end{aligned}$$

The error arising between this finite element solution of problem (1.5) - (1.7) and the exterior Stokes flow (u,π) may be estimated like this:

Theorem 1.5 *(see [2]): There are constants $C_1 > 0$, $C_2 > S$, $C_3 \in (0,S)$ depending on Ω, S and γ such that it holds for $h \in (0,C_3)$, $R \in (C_2,\infty)$:*

$$\big\| \, v(h,R) \,|\, \Omega_S \;-\; u \,|\, \Omega_S \, \big\|_2 \;\leq\; C_1 \cdot \big(R^{-3/2} \cdot \mathcal{M}(\epsilon,R) \;+\; h/R \;+\; h^2 \big), \qquad (1.9)$$

with γ from (1.8), $\mathcal{M}(\epsilon,R)$ and S from Theorem 1.3, and with the functions $v(h,R)$, $\varrho(h,R)$ introduced in Theorem 1.4.

We remark that the term $R^{-3/2} \cdot \mathcal{M}(\epsilon,R)$ corresponds to the truncation error (Theorem 1.3). The contribution h/R is due to the fact that we decompose $P(h,R)$

instead of Ω_R, and the term h^2 represents the usual discretization error. Obviously the latter component of the error is estimated in the best possible way in (1.9) because over each element, our finite element spaces consist of P1 and bubble functions, with the bubbles not contributing anything to accuracy. The estimate of the truncation error also seems to be best possible, as explained above. We further point out that if the truncation and discretization error are comparable, that is, if we only admit values of h and R such that $h^2 \,/\, R^{-3/2} \in [\gamma_1, \gamma_2]$ for fixed numbers γ_1 and γ_2 with $\gamma_1 < \gamma_2$, then the term h/R on the left hand side of (1.9) contributes less to the total error than the sum $R^{-3/2} + h^2$. So the error estimate stated in (1.9) should be optimal.

2. Test computations.

Our finite element method leads to an algebraic system of the form

$$\begin{pmatrix} A & B^T \\ B & 0 \end{pmatrix} \cdot \begin{pmatrix} V \\ P \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix},$$

with $A \in \mathbb{R}^{N\times N}$ and $B \in \mathbb{R}^{M\times N}$ for some integers M, $N \in \mathbb{N}$, where A is symmetric and positive definite, and $kernel\, B^T = \{0\}$. Of course, such an algebraic system is standard in the context of the Stokes equations. In order to solve it, we implemented the Uzawa cojugate gradient method described in [5, p. 289-294]. For the inner iteration which is part of this algorithm, we used a preconditioned cg-method, with the inverse of the diagonal of A as preconditioner. As should be expected, convergence of the outer iteration turned out to be slow due to persistence of low modes of the error. We plan to improve this solver. But even in its present form, it allows us to exploit the sparsity of the mass matrices arising with our finite element methods.

We started our computations with a truncating sphere having radius $4\cdot\sqrt{3}$ and with a mesh having 79 nodes and 312 tetrahedrons. We refined this mesh twice, and with each refinement step, we doubled the radius of the truncating sphere. The number of gridpoints and tetrahedrons obtained in this way are listed in Table 1, where we additionally indicate the number of unknowns arising in the resulting algebraic system. More details of our mesh generation process may be found in [2]. Up to now we cannot handle the third refinement step indicated in Table 1 because our solver is too slow.

Table 1

# refinement steps	# nodes	# tetrahedrons	# unknowns
0	79	312	1 252
1	803	3 648	14 156
2	8 141	38 400	147 764
3	79 849	380 928	1 462 180

According to the error estimate stated in Theorem 1.5, the truncation error decreases with each refinement step by a factor of $2^{-3/2}\cdot\alpha$, with $\alpha = 1$ if $\epsilon > 0$ in (1.4), and $\alpha = 1 + \ln 2$ in the case $\epsilon = 0$. Thus the truncation error decreases with each refinement step by a factor of about 0.35 in the first case and with a rate of about 0.60 in the second one. Theorem 1.5 predicts a decrease by a factor 0.25 for the other components of the error. This means that with our choice of meshes, the truncation and discretization errors are not balanced, and the truncation error predominates, so the total error should decrease with each refinement step by a factor of about 0.35 and 0.60, respectively.

As a first test example, we take $\Omega = \emptyset$ and $S = 4.0$, and we choose f in such a way that the exterior flow (u, π) is given by

$$u(x) = 10 \cdot \mathrm{rot}\left(e^{-|x|^2}, e^{-|x|^2}, e^{-|x|^2}\right), \quad \pi(x) = e^{-|x|^2}, \quad \text{for } x \in \mathbb{R}^3.$$

The results obtained with this setup are presented in Table 2. We see that the error decreases with a rate which is rather close to or less than the predicted one.

Table 2

# refinement steps	error rate	rate of decrease of error	# inner iterations	# outer iterations
0	20.42		20	32
1	8.04	0.39	57	227
2	2.10	0.26	118	1089

As a second test example, we again choose $\Omega = \emptyset$, $S = 4.0$, and we define f in such a way that the exterior flow (u, π) is given by the following functions u and π :

$$u(x) = \mathrm{rot}\left(g(x), g(x), g(x)\right) \quad \text{for } x \in \mathbb{R}^3 \text{ with } |x| > 1,$$

where

$$g(x) := (|x| - 1)^4 \cdot |x|^{-4} \quad \text{for } x \in \mathbb{R}^3 \setminus B_1.$$

For $x \in \overline{B_1}$, we put $u(x) := 0$ and $\pi(x) := 0$. This function u decays much slower then the corresponding function in the preceding example. In fact, we now have

$$\left|u(x)\right| = O(|x|^{-1}), \qquad \left|f(x)\right| = O(|x|^{-4}) \qquad \text{for } |x| \to \infty.$$

The results which our computations yielded for such an exterior flow are listed in Table 3. It shows in particular that the error decays at least with the predicted rate. Note that assumption (1.4) is now satisfied with $\epsilon = 0$.

Table 3

# refinement steps	error rate	rate of decrease of error	# inner iterations	# outer iterations
0	5.69		20	18
1	3.12	0.55	55	145
2	1.11	0.33	136	654

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