CORRECTION Exercices Chapitre 0 - Rappels sur les polynômes et fractions algébriques.

Exercice 1

1. $a^3 \times a^2 = a^{3+2} = a^5$
2. $\frac{a^5}{a^3} = a^{5-3} = a^2$
3. $(a \times b)^3 = a^3 b^3$
4. $\left(\frac{a}{b}\right)^4 = \frac{a^4}{b^4}$
5. $(a^3)^2 = a^{3\times 2} = a^6$

Exercice 2

1. $x^3 \times x^2 = x^{3+2} = x^5$
2. $3a \times 2a^2 = 3 \times 2 \times a^{1+2} = 6a^3$
3. $-3x^2 \times 4x^3 = (-3) \times 4 \times x^{2+3} = -12x^5$
4. $2x^2 y^3 \times 3xy^4 = 2 \times 3 \times x^{2+1} y^{3+4} = 6x^3 y^7$
5. $8a^4 b^3 c^2 \times 2ab^2 c^3 = 8 \times 2 \times a^{4+1} b^{3+2} c^{2+3} = 16a^5 b^5 c^5$
6. $\frac{2}{5}a^3 b^2 \times \frac{-5}{8}a^2 b = \frac{2}{5} \times \frac{-5}{8} \times a^{3+2} b^{2+1} = -\frac{1}{4}a^5 b^3$

Exercice 3

1. $a^5 \div a^2 = a^{5-2} = a^3$
2. $12a^6 \div 4a^4 = \frac{12}{4} \times a^{6-4} = 3a^2$
3. $15x^5 \div -3x^5 = \frac{15}{-3} \times x^{5-5} = -5x^0 = -5$
4. $\frac{-12a^2}{3a} = \frac{-12}{3} \times a^{2-1} = -4a$
5. $\frac{-36a^4 b^3}{18a^2 b^2} = \frac{-36}{18} \times a^{4-2} b^{3-2} = -2a^2 b$
6. $\frac{16x^5 y^2}{-32x^4} = \frac{16}{-32} \times x^{5-4} y^2 = -\frac{1}{2}xy^2$

Exercice 4

1. $(2a)^3 = 2^3 \times a^3 = 8a^3$
2. $(5a^3)^2 = 5^2 \times a^{3\times 2} = 25a^6$
3. $(2a^3)^2 \times 3a = 2^2 \times a^{3\times 2} \times 3a = 4 \times 3 \times a^{6+1} = 12a^7$
4. $(a^2 b^3)^2 \times (5a)^2 = a^4 b^6 \times 25a^2 = 25a^6 b^6$
5. $(3a^4)^2 \times (2a)^4 = 9a^8 \times 16a^4 = 144a^{12}$
6. $(2a^3 b^2)^4 \times (3a^2 b)^2 = 16a^{12} b^8 \times 9a^4 b^2 = 144a^{16} b^{10}$

Exercice 5

1. $\left(\frac{2}{3}a\right)^2 = \frac{4}{9}a^2$
2. $\left(\frac{3a^2}{b}\right)^3 = \frac{27a^6}{b^3}$
3. $\left(\frac{-3a}{b^2}\right)^3 = \frac{-27a^3}{b^6}$
4. $\left(\frac{3x^2 y}{2}\right)^4 = \frac{81x^8 y^4}{16}$
5. $\left(\frac{-5a^2 b^3}{c^4}\right)^2 = \frac{25a^4 b^6}{c^8}$
6. $\left(\frac{-a^4}{2bc^2}\right)^5 = \frac{-a^{20}}{32b^5 c^{10}}$

1. $\left(\frac{2a^2}{3b^3}\right)^2 \times \left(\frac{3a}{b}\right)^3 = \frac{4a^4}{9b^6} \times \frac{27a^3}{b^3} = \frac{108a^7}{9b^9} = \frac{12a^7}{b^9}$

2. $\left(\frac{-3x^2}{5y}\right)^3 \times \left(\frac{x^3}{2y^2}\right)^2 = \frac{-27x^6}{125y^3} \times \frac{x^6}{4y^4} = \frac{-27x^{12}}{500y^7}$

3. $\left(\frac{10a^2b^6}{3a}\right) \times \left(\frac{3a^4}{5ab^5}\right)^2 = \frac{10a^2b^6}{3a} \times \frac{9a^8}{25a^2b^{10}} = \frac{90a^{10}b^6}{75a^3b^{10}} = \frac{6a^7}{5b^4}$

4. $\frac{5a^3}{(b^2)^3} \times \left(\frac{b^3}{2a}\right)^2 = \frac{5a^3}{b^6} \times \frac{b^6}{4a^2} = \frac{5a^3}{4a^2} = \frac{5}{4}a$

Exercice 7

1. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

2. $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$

3. $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

4. $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

5. $10^{-1} = \frac{1}{10}$

6. $(0,01)^{-2} = \left(\frac{1}{100}\right)^{-2} = 100^2 = 10000$

7. $(-0,1)^{-2} = \left(-\frac{1}{10}\right)^{-2} = (-10)^2 = 100$

8. $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

Exercice 8

1. $2^3 \times 2^{-4} \times 2^2 = 2^{3-4+2} = 2^1 = 2$

2. $(5^{-2})^3 \times 5^8 = 5^{-6} \times 5^8 = 5^{-6+8} = 5^2 = 25$

3. $(2^{-3})^2 \times (2^{-3})^{-4} = 2^{-6} \times 2^{12} = 2^6 = 64$

4. $(a^2b^{-4})^{-1} \times (a^{-3}b)^{-2} = (a^{-2}b^4) \times (a^6b^{-2}) = a^4b^2$

5. $(-2a^2b^{-1})^{-2} \times (2a^{-1}b)^2 = \left(\left(-\frac{1}{2}\right)^2 a^{-4}b^2\right) \times (4a^{-2}b^2) = a^{-6}b^4$

6. $\left(\frac{2a^{-3}}{b^{-2}}\right)^{-2} = \frac{\left(\frac{1}{2}\right)^2 a^6}{b^4} = \frac{a^6}{4b^4} = \frac{1}{4}a^6b^{-4}$

7. $\frac{12a^{-3}b^2}{4a^{-4}b^{-1}} = 3a^{-3-(-4)}b^{2-(-1)} = 3ab^3$

8. $\left(\frac{-2a^{-1}b^2}{3a^{-2}b}\right)^2 = \frac{4a^{-2}b^4}{9a^{-4}b^2} = \frac{4}{9}a^2b^2$

9. $\left(\frac{2a^4b^{-2}}{a^{-3}b}\right)^{-2} = \left(\frac{a^{-3}b}{2a^4b^{-2}}\right)^2 = \frac{a^{-6}b^2}{4a^8b^{-4}} = \frac{1}{4}a^{-14}b^6$

Exercice 9

1. $(2a^4b^{-2})^{-1} = \frac{1}{2a^4b^{-2}} = \frac{1}{2}a^{-4}b^2$

2. $(a^2b^{-4})^{-1} \times (a^{-3}b)^{-2} = \frac{1}{a^2b^{-4}} \times \left(\frac{1}{a^{-3}b}\right)^2 = \frac{1}{a^2b^{-4}} \times \frac{1}{a^{-6}b^2} = \frac{1}{a^{-4}b^{-2}} = a^4b^2$

3. $\left(\frac{a}{b^{-2}}\right)^{-1} = \frac{b^{-2}}{a} = b^{-2}a^{-1}$

4. $(2ab)^{-2} \times 4a^2b^3 = \left(\frac{1}{2ab}\right)^2 \times 4a^2b^3 = \frac{4a^2b^3}{4a^2b^2} = b$

5. $(-3a^2b^{-3})^2 \times (2a^{-1}b^2)^{-2} = (9a^4b^{-6}) \times \left(\frac{1}{2a^{-1}b^2}\right)^2 = \frac{9a^4b^{-6}}{4a^{-2}b^4} = \frac{9}{4}a^6b^{-10}$

6. $(-a^2b^{-1})^3 \div (ab^{-2})^2 = \frac{-a^6b^{-3}}{a^2b^{-4}} = -a^4b$

Exercice 10

1. $\sqrt{25} = 5$

2. $\sqrt{-25}$ n'existe pas dans $\mathbb{R}$

3. $\sqrt[4]{16} = 2$

4. $\sqrt[4]{-16}$ n'existe pas dans $\mathbb{R}$

5. $\sqrt[3]{64} = 4$

6. $\sqrt[3]{-64} = -4$

7. $\sqrt[4]{256} = 4$

8. $\sqrt{256} = 16$

Exercice 11 Calculer à l'aide de la calculatrice (arrondir le résultat au centième près) :

1. $\sqrt{2} = 1,41$

2. $\sqrt{3} = 1,73$

3. $\sqrt{5} = 2,25$

4. $\sqrt[3]{2} = 1,26$

5. $\sqrt[3]{3} = 1,44$

6. $\sqrt[3]{5} = 1,71$

7. $\sqrt{10} = 3,16$

8. $\sqrt[4]{10} = 1,78$

Exercice 12

1. $\sqrt{x-2}$ est défini pour $x \in \mathbb{R}$ tel que $x - 2 \geq 0 \Leftrightarrow x \geq 2$.

2. $\sqrt[3]{x-2}$ est défini pour tout $x \in \mathbb{R}$.

3. $\sqrt{-2x+1}$ est défini pour $x \in \mathbb{R}$ tel que $-2x + 1 \geq 0 \Leftrightarrow x \leq \frac{1}{2}$.

4. $\frac{1}{\sqrt{2x-6}}$ est défini pour $x \in \mathbb{R}$ tel que $2x - 6 > 0 \Leftrightarrow x > 3$.

Exercice 13

1. Si x est positif, $\sqrt{x^2} = x$.

2. Si x est négatif, $\sqrt{x^2} = -x$.

Exercice 14

1. $25^{\frac{1}{2}} = 5$

2. $(-16)^{\frac{1}{2}}$ n'existe pas dans $\mathbb{R}$

3. $729^{\frac{1}{3}} = 9$

4. $(-729)^{\frac{1}{3}} = -9$

5. $(4^{\frac{1}{2}})^3 = 2^3 = 8$

6. $(4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} = 8$

7. $\left(\frac{-8}{27}\right)^{\frac{1}{3}} = \frac{-2}{3}$

8. $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$

9. $(16^{-1})^{\frac{1}{4}} = \left(\frac{1}{16}\right)^{\frac{1}{4}} = \frac{1}{2}$

Exercice 15

1. $16^{\frac{3}{2}} = (16^{\frac{1}{2}})^3 = 4^3 = 64$

2. $(-27)^{\frac{2}{3}} = (-27^{\frac{1}{3}})^2 = (-3)^2 = 9$

3. $9^{-\frac{3}{2}} = \left(\frac{1}{9}\right)^{\frac{3}{2}} = \left(\left(\frac{1}{9}\right)^{\frac{1}{2}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

4. $4^{1,5} = 4^{\frac{3}{2}} = (4^{\frac{1}{2}})^3 = 2^3 = 8$

5. $25^{-2,5} = 25^{-\frac{5}{2}} = \left(\frac{1}{25}\right)^{\frac{5}{2}} = \left(\left(\frac{1}{25}\right)^{\frac{1}{2}}\right)^5 = \left(\frac{1}{5}\right)^5 = \frac{1}{3125}$

6. $\left(\frac{16}{25}\right)^{\frac{3}{2}} = \left(\left(\frac{16}{25}\right)^{\frac{1}{2}}\right)^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

7. $(-9)^{\frac{3}{2}}$ n'existe pas dans $\mathbb{R}$

8. $\left(-\frac{32}{243}\right)^{-\frac{2}{5}} = \left(-\frac{243}{32}\right)^{\frac{2}{5}} = \left(\left(-\frac{243}{32}\right)^{\frac{1}{5}}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

9. $\left(\frac{1}{4}\right)^{-0,5} = \left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}} = 2$

Exercice 16

1. $a^{\frac{1}{2}} \times a^{\frac{2}{3}} = a^{\frac{1}{2}+\frac{2}{3}} = a^{\frac{1\times3+2\times2}{2\times3}} = a^{\frac{7}{6}}$

2. $a^{-\frac{1}{2}} \times a^{\frac{1}{3}} = a^{-\frac{1}{2}+\frac{1}{3}} = a^{\frac{(-1)\times3+1\times2}{2\times3}} = a^{-\frac{1}{6}}$

3. $a^{\frac{2}{3}} \div a^{\frac{1}{4}} = a^{\frac{2}{3}} \times a^{-\frac{1}{4}} = a^{\frac{2}{3}-\frac{1}{4}} = a^{\frac{2\times4-3\times1}{3\times4}} = a^{\frac{5}{12}}$

4. $\left(a^{\frac{3}{2}}\right)^2 = a^3$

5. $\left(a^{\frac{2}{3}}\right)^{\frac{3}{4}} \times \left(a^{\frac{1}{4}}\right)^2 = a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$

6. $\left(a^{\frac{1}{2}}\right)^3 \div \left(a^{\frac{1}{3}}\right)^2 = \left(a^{\frac{1}{2}}\right)^3 \times \left(a^{\frac{1}{3}}\right)^{-2} = a^{\frac{3}{2}} \times a^{-\frac{2}{3}} = a^{\frac{3}{2}-\frac{2}{3}} = a^{\frac{5}{6}}$

Exercice 17

1. $a^{-\frac{2}{3}} \times a^{-\frac{3}{4}} = a^{-\frac{2}{3}-\frac{3}{4}} = a^{-\frac{17}{12}}$

2. $a^{\frac{5}{2}} \div a^{-\frac{5}{3}} = a^{\frac{5}{2}} \times a^{\frac{5}{3}} = a^{\frac{25}{6}}$

3. $a^{-\frac{3}{2}} \times a^{\frac{5}{6}} = a^{-\frac{4}{6}} = a^{-\frac{2}{3}}$

4. $a^{-\frac{1}{2}} \div a^{-\frac{1}{3}} = a^{-\frac{1}{2}} \times a^{\frac{1}{3}} = a^{-\frac{1}{6}}$

5. $\left(a^{\frac{1}{2}}\right)^{-3} = a^{-\frac{3}{2}}$

6. $(a^{-\frac{2}{3}})^{-2} \times (a^{-\frac{1}{3}})^{-5} = a^{\frac{4}{3}} \times a^{\frac{5}{3}} = a^{\frac{9}{3}} = a^3$

Exercice 18

1. $(a^2 b^3)^{\frac{1}{6}} = a^{\frac{2}{6}} b^{\frac{3}{6}} = a^{\frac{1}{3}} b^{\frac{1}{2}}$

2. $(a^{-2} b^3)^{-\frac{1}{2}} = a b^{-\frac{3}{2}}$

3. $\left(\frac{a^3}{b^2}\right)^{\frac{1}{6}} = \frac{a^{\frac{3}{6}}}{b^{\frac{2}{6}}} = \frac{a^{\frac{1}{2}}}{b^{\frac{1}{3}}} = a^{\frac{1}{2}} b^{-\frac{1}{3}}$

4. $\left(\frac{a^3 b^2}{c^4}\right)^{\frac{1}{12}} = \frac{a^{\frac{3}{12}} b^{\frac{2}{12}}}{c^{\frac{4}{12}}} = \frac{a^{\frac{1}{4}} b^{\frac{1}{6}}}{c^{\frac{1}{3}}} = a^{\frac{1}{4}} b^{\frac{1}{6}} c^{-\frac{1}{3}}$

5. $(a^{-\frac{2}{3}} \div a^{\frac{3}{4}})^{-2} = \frac{a^{\frac{4}{3}}}{a^{-\frac{6}{4}}} = a^{\frac{4}{3}} \times a^{\frac{6}{4}} = a^{\frac{4}{3}+\frac{3}{2}} = a^{\frac{17}{6}}$

6. $[(a^2 b^{-3})^{\frac{2}{3}}]^6 = (a^{\frac{4}{3}} b^{-2})^6 = a^8 b^{-12}$

Exercice 19

1. $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$

2. $\sqrt[3]{9}\sqrt[3]{3} = \sqrt[3]{27} = 3$

3. $\frac{\sqrt{32}}{\sqrt{2}} = \frac{\sqrt{16\times2}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{2}} = 4$

4. $\sqrt[3]{\sqrt{729}} = \sqrt[3]{27} = 3$

5. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \frac{\sqrt[3]{8\times2}}{\sqrt[3]{2}} = \frac{2\sqrt[3]{2}}{\sqrt[3]{2}} = 2$

6. $\sqrt[3]{125^2} = (\sqrt[3]{125})^2 = 5^2 = 25$

Exercice 20

1. $2\sqrt{12} \times 5\sqrt{3} = 10\sqrt{36} = 60$

2. $-2\sqrt[4]{2} \times 3\sqrt[4]{8} = -6\sqrt[4]{16} = -12$

3. $(2\sqrt{5})^2 = 4 \times 5 = 20$

4. $\frac{10\sqrt[3]{81}}{5\sqrt[3]{3}} = \frac{2\sqrt[3]{27\times3}}{\sqrt[3]{3}} = \frac{2\times3\sqrt[3]{3}}{\sqrt[3]{3}} = 6$

5. $(2\sqrt[3]{5})^3 = 8 \times 5 = 40$

6. $\frac{3}{4}\sqrt{\frac{16}{9}} = \frac{3}{4} \times \frac{4}{3} = 1$

Exercice 21

1. $3\sqrt[3]{18} \times 2\sqrt[3]{12} = 6\sqrt[3]{216} = 6 \times 6 = 36$

2. $2\sqrt[3]{4} \times 3\sqrt[3]{250} = 6\sqrt[3]{1000} = 6 \times 10 = 60$

3. $\left(\sqrt[3]{\frac{125}{8}}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

4. $\left(\frac{3}{4}\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{3}{4} \times \frac{2}{3}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

5. $(2\sqrt{3})^2 \times (3\sqrt{2})^2 = 12 \times 18 = 216$

6. $\frac{12\sqrt{50}}{5\sqrt{2}} = \frac{12\sqrt{2 \times 25}}{5\sqrt{2}} = \frac{60\sqrt{2}}{5\sqrt{2}} = 12$

Exercice 22

1. $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

2. $\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}$

3. $\sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}$

4. $\sqrt{720} = \sqrt{144 \times 5} = 12\sqrt{5}$

5. $3\sqrt{500} = 3\sqrt{100 \times 5} = 30\sqrt{5}$

6. $\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

7. $\sqrt[3]{250} = \sqrt[3]{125 \times 2} = 5\sqrt{2}$

8. $\sqrt[4]{80} = \sqrt[4]{16 \times 5} = 2\sqrt[4]{5}$

9. $\frac{4}{5}\sqrt{1200} = \frac{4}{5}\sqrt{400 \times 3} = \frac{80}{5}\sqrt{3} = 16\sqrt{3}$

10. $\sqrt[3]{2000} = \sqrt[3]{1000 \times 2} = 10\sqrt[3]{2}$

11. $\frac{2}{3}\sqrt[4]{162} = \frac{2}{3}\sqrt[4]{81 \times 2} = \frac{2}{3} \times 3\sqrt[4]{2} = 2\sqrt[4]{2}$

12. $-7\sqrt[4]{160} = -7\sqrt[4]{16 \times 10} = -7 \times 2\sqrt[14]{10} = -14\sqrt[4]{10}$

Exercice 23

1. $5x$ est un monôme en x de degré 1,

2. $\frac{5}{x}$ n'est pas un monôme car $\frac{5}{x} = 5x^{-1}$ et $-1 \notin \mathbb{N}$,

3. $3x^{-2}$ n'est pas un monôme car $-2 \notin \mathbb{N}$,

4. $2x^{\frac{2}{3}}$ n'est pas un monôme car $\frac{2}{3} \notin \mathbb{N}$,

5. -1 est un monôme en x de degré 0.

Exercice 24

1. Le périmètre d'un carré de côté x : $4x$

2. L'aire d'un disque de rayon r : πr^2

3. Le volume d'une sphère de rayon r : $\frac{1}{3}\pi r^3$

4. L'aire latérale d'un cylindre droit de rayon r et de hauteur h : $2\pi rh$

5. Le volume d'un cylindre droit de rayon r et de hauteur h : $\pi r^2 h$

Exercice 25

1. Si $P(x) = -3x^2$, $P(-5) = -3(-5)^2 = -75$.

2. Si $P(x,y) = -2x^2y^3$, $P(3,-2) = -2(3)^2(-2)^3 = 144$

3. Si $P(x,y) = 3x^2y^3$, $P\left(\frac{2}{3}, -\frac{3}{2}\right) = 3\left(\frac{2}{3}\right)^2\left(-\frac{3}{2}\right)^3 = 3 \times \frac{4}{9} \times \left(\frac{-27}{8}\right) = -\frac{9}{2}$

4. Si $P(x,y) = -\frac{2}{3}xy^2$, $P\left(\frac{3}{4}, -\frac{1}{2}\right) = -\frac{2}{3}\left(\frac{3}{4}\right)\left(-\frac{1}{2}\right)^2 = -\frac{1}{8}$

Exercice 26

1. $-3x$ n'est pas semblable aux autres puisque ce monôme n'est pas de degré 2.

2. $3xy^2$ n'est pas semblable aux autres puisque ce monôme n'est pas de la forme kx^2y.

Exercice 27

1. $2x^3 - 5x^3 + 7x^3 = (2 - 5 + 7)x^3 = 4x^3$

2. $4x^2y - 6x^2y + x^2y = (4 - 6 + 1)x^2y = -x^2y$

3. $\frac{3}{4}x^2 + \frac{2}{3}x^2 - x^2 = \left(\frac{3}{4} + \frac{2}{3} - 1\right)x^2 = \frac{5}{12}x^2$

4. $-\frac{2}{3}xy^2 + \frac{3}{4}xy^2 - \frac{5}{6}xy^2 = \left(-\frac{2}{3} + \frac{3}{4} - \frac{5}{6}\right)xy^2 = -\frac{9}{12}xy^2 = -\frac{3}{4}xy^2$

1. $-3x^2 \times 4x^3 = -12x^5$
2. $2x^2y^3 \times -3xy^2 = -6x^3y^5$
3. $-17x^2 \times -3x = 51x^3$
4. $-7x^2y \times 5x^2y^2 = -35x^4y^3$
5. $3x^2y \times -5xy \times -2xy^2 = 30x^4y^4$
6. $20x^2y^2 \times -0,5x \times -1,2y^2 = 12x^3y^4$
7. $\frac{3}{4}x^2y \times \frac{2}{5}xy^2 \times \frac{10}{9}x = \frac{60}{180}x^4y^3 = \frac{1}{3}x^4y^3$
8. $\frac{-3}{5}x^2y^3 \times \frac{2}{3}xy \times \frac{-5}{2}xy^2 = \frac{30}{30}x^4y^6 = x^4y^6$

Exercice 29

1. $(3x^2y)^2 = 9x^4y^2$
2. $(-2xy^3)^2 = 4x^2y^6$
3. $(-2x^2y^3)^3 = -8x^6y^9$
4. $\left(-\frac{3}{4}x^2y^3\right)^2 = \frac{9}{16}x^4y^6$
5. $3x^2(-2x)^3 = -24x^5$
6. $(-2x^2)^2 \times (3x^3)^2 = 4x^4 \times 9x^6 = 36x^{10}$

Exercice 30

1. $-12x^4 \div 3x^6 = \frac{-12x^4}{3x^6} = \frac{-4}{x^2}$ n'est pas un monôme,
2. $18x^6 \div 12x^4 = \frac{18x^6}{12x^4} = \frac{3x^2}{2}$ est un monôme,
3. $18x^6y^4 \div 9x^2y^2 = \frac{18x^6y^4}{9x^2y^2} = 2x^4y^2$ est un monôme,
4. $-12x^2y^4 \div 6x^3y = \frac{-12x^2y^4}{6x^3y} = \frac{-2y^3}{x}$ n'est pas un monôme,
5. $(-5x^3)^2 \div 10x^4 = \frac{25x^6}{10x^4} = \frac{5x^2}{2}$ est un monôme,
6. $(4x^2y^3)^3 \div (2xy^2)^4 = \frac{64x^6y^9}{16x^4y^8} = 4x^2y$ est un monôme.

Exercice 31

1. La somme de deux monômes n'est pas toujours un monôme, par exemple $x + x^2$ n'est pas un monôme.
2. Le produit de deux monômes est toujours un monôme. En effet, soient ax^p ($p \in \mathbb{N}$) et bx^q ($q \in \mathbb{N}$) deux monômes, on a alors $ax^p \times bx^q = abx^{p+q}$ qui est un monôme car $p + q \in \mathbb{N}$.
3. Le quotient de deux monômes n'est pas toujours un monôme, par exemple $\frac{x}{x^2}$ n'est pas un monôme.

Exercice 32

1. $x\sqrt{2} + 3x^2 - 1$ est un polynôme, il peut se réécrire sous la forme $3x^2 + \sqrt{2}x - 1$ et est de degré 2,
2. $2x^2 + 2\sqrt{x}$ n'est pas un polynôme car $2\sqrt{x} = 2x^{\frac{1}{2}}$ n'est pas un monôme,
3. $3y - 2y^3 + \frac{1}{2}y^4 - 4y^2 + 1$ est un polynôme, il peut se réécrire sous la forme $\frac{1}{2}y^4 - 2y^3 - 4y^2 + 3y + 1$ et est de degré 4,
4. $x^{-1} + 5x + x^2$ n'est pas un polynôme car x^{-1} n'est pas un monôme.

Exercice 33

1. $P(x) = 3x^2 + 2x - 5x^2 - 3x + 1 = (3-5)x^2 + (2-3)x + 1 = -2x^2 - x + 1$ est de degré 2,
2. $P(x,y) = 3x^3y - 2xy^2 + 4x^3y - xy^2 = (3+4)x^3y + (-2-1)xy^2 = 7x^3y - 3xy^2$ est de degré (total) 4,
3. $P(z) = 4z^3 - 5z^2 + 8z^3 - z^2 + 4z - 5 + 6z^2 - 12z^3 = (4+8-12)z^3 + (-5-1+6)z^2 + 4z - 5 = 4z - 5$ est de degré 1,
4. $P(x) = \frac{3}{2}x^2 + 5x^3 - \frac{2}{3}x^2 - \frac{3}{2}x^3 + \frac{3}{4}x - \frac{5}{2}x = (5-\frac{3}{2})x^3 + (\frac{3}{2}-\frac{2}{3})x^2 + (\frac{3}{4}-\frac{5}{2})x = \frac{7}{2}x^3 + \frac{5}{6}x^2 - \frac{7}{4}x$ est de degré 3.

Exercice 34

1. $P(-2) = 3(-2)^2 + 5(-2) = 12 - 10 = 2$
2. $P(0) = (0)^2 - 5(0) + 3 = 3$
3. $P(-1,5) = P(-\frac{3}{2}) = 3(-\frac{3}{2})^2 + 2(-\frac{3}{2}) - 5 = -\frac{5}{4}$
4. $P(-\frac{3}{2}) = 2(-\frac{3}{2})^2 - 7(-\frac{3}{2}) - 15 = 0$

Exercice 35

1. $P(2,3) = 3(2)^2(3) - 2(2)(3)^2 + (2)(3) - 2 = 36 - 36 + 6 - 2 = 4$
2. $P(-3,1) = 3(-3)^2(1) - 2(-3)(1)^2 + (-3)(1) - 2 = 28$
3. $P(-1,-2) = 3(-1)^2(-2) - 2(-1)(-2)^2 + (-1)(-2) - 2 = 2$
4. $P\left(-\frac{2}{3}, \frac{3}{2}\right) = 3(-\frac{2}{3})^2(\frac{3}{2}) - 2(-\frac{2}{3})(\frac{3}{2})^2 + (-\frac{2}{3})(\frac{3}{2}) - 2 = 2$

Exercice 36

1. Comme $P(-1) = 2(-1)^2 - (-1) - 6 = -3 \neq 0$, -1 n'est pas un zéro (une racine) de $P(x)$.
2. Comme $P(2) = 2(2)^2 - (2) - 6 = 0$, 2 est un zéro de $P(x)$.
3. Comme $P(0) = 2(0)^2 - (0) - 6 = -6$, 0 n'est pas un zéro de $P(x)$.
4. Comme $P(-\frac{3}{2}) = 2(-\frac{3}{2})^2 - (-\frac{3}{2}) - 6 = 0$, $-\frac{3}{2}$ est un zéro de $P(x)$.

Exercice 37

1. $P(x) = 0 \Leftrightarrow 3x - 2 = 0 \Leftrightarrow x = \frac{2}{3}$,
2. $P(y) = 0 \Leftrightarrow -2y + 5 = 0 \Leftrightarrow y = \frac{5}{2}$,
3. $P(z) = 0 \Leftrightarrow -2z = 0 \Leftrightarrow z = 0$,
4. $P(x) = 0 \Leftrightarrow -3x + \frac{1}{2} = 0 \Leftrightarrow x = \frac{1}{6}$,
5. $P(y) = 0 \Leftrightarrow -\frac{2}{3}y + 4 = 0 \Leftrightarrow y = 6$,
6. $P(z) = 0 \Leftrightarrow -\frac{2}{5}z + \frac{3}{4} = 0 \Leftrightarrow z = \frac{15}{8}$.

Exercice 38 Il suffit d'évaluer $d(t) = 4,9t^2 + 3t$ en $t = 3$: comme $d(3) = 4,9(3)^2 + 3(3) = 53,1$, le pont se trouve à $53,1$m de hauteur.

Exercice 39

1. $P(x) + Q(x) + R(x) = (3x^2 - 2x + 1) + (-x^2 - 3x + 2) + (-2x + 5) = 2x^2 - 7x + 8$
2. $P(x) - Q(x) + R(x) = (3x^2 - 2x + 1) - (-x^2 - 3x + 2) + (-2x + 5) = 4x^2 - x + 4$
3. $P(x) - Q(x) - R(x) = (3x^2 - 2x + 1) - (-x^2 - 3x + 2) - (-2x + 5) = 4x^2 + 3x - 6$
4. $-P(x) + Q(x) - R(x) = -(3x^2 - 2x + 1) + (-x^2 - 3x + 2) - (-2x + 5) = -4x^2 + x - 4$

Exercice 40

1. $P(x) + Q(x) + R(x) = (\frac{2}{3}x^2 - \frac{3}{2}x + 1) + (\frac{3}{2}x^2 + \frac{5}{6}x - \frac{1}{3}) + (\frac{2}{3}x - \frac{1}{6}) = \frac{13}{6}x^2 + \frac{1}{2}$
2. $P(x) - Q(x) + R(x) = (\frac{2}{3}x^2 - \frac{3}{2}x + 1) - (\frac{3}{2}x^2 + \frac{5}{6}x - \frac{1}{3}) + (\frac{2}{3}x - \frac{1}{6}) = \frac{-5}{6}x^2 - \frac{5}{3}x + \frac{7}{6}$
3. $P(x) - Q(x) - R(x) = (\frac{2}{3}x^2 - \frac{3}{2}x + 1) - (\frac{3}{2}x^2 + \frac{5}{6}x - \frac{1}{3}) - (\frac{2}{3}x - \frac{1}{6}) = \frac{-5}{6}x^2 - 3x + \frac{3}{2}$
4. $-P(x) + Q(x) - R(x) = -(\frac{2}{3}x^2 - \frac{3}{2}x + 1) + (\frac{3}{2}x^2 + \frac{5}{6}x - \frac{1}{3}) - (\frac{2}{3}x - \frac{1}{6}) = \frac{5}{6}x^2 + \frac{5}{3}x - \frac{7}{6}$

Exercice 41

1. $(4x^2 - 8x + 1) - (2x^2 - 3x + 5) = (4-2)x^2 + (-8-(-3))x + (1-5) = 2x^2 - 5x - 4$
2. $(3x^2y - 2xy^2 + 3xy) + (2x^2y + 3x^2y - 5xy) = (3+2+3)x^2y - 2xy^2 + (3-5)xy = 8x^2y - 2xy^2 - 2xy$
3. $(3a^2b - 5ab^2) - (2a^2b + 3ab^2) = (3-2)a^2b + (-5-3)ab^2 = a^2b - 8ab^2$
4. $(3x^2y - 2xy + 4xy^2) - (-3xy^2 + 4xy) + 2x^2y = (3+2)x^2y + (-2-4)xy + (4-(-3))xy^2 = 5x^2y - 6xy + 7xy^2$
5. $(3x^3 - 5x^2 - 4x - 1) - [(x^3 - 5x^2) - (x^2 - 4x + 1)] = (3-1)x^3 + (-5-(-5)-1)x^2 + (-4-(-(-4))x - 1 - (-1) = 2x^3 - x^2 - 8x$

1. $3x^2(2x - 5) = 6x^3 - 15x^2$
2. $-3y(y^2 - 2y) = -3y^3 + 6y^2$
3. $-2x^2(3xy^2 + 5x^2y) = -6x^3y^2 - 10x^4y$
4. $(2xy - 5x)(-3x^2y) = -6x^3y^2 + 15x^3y$
5. $\frac{3}{4}x^2\left(\frac{2}{3}x - 8x^2\right) = \frac{1}{2}x^3 - 6x^4$
6. $(4x^2 - 8x + 12)\left(-\frac{3}{4}x\right) = -3x^3 + 6x^2 - 9x$

1. $(x + 3)(x - 2) = x^2 + (3 - 2)x - 6 = x^2 + x - 6$
2. $(x - 5)(3 - x) = -x^2 + (3 + 5)x - 15 = -x^2 + 8x - 15$
3. $(2a + b)(3a - 2b) = 6a^2 + (3 - 4)ab - 2b^2 = 6a^2 - ab - 2b^2$
4. $(5 - 2x)(3x - 4) = -6x^2 + (15 + 8)x - 20 = -6x^2 + 23x - 20$
5. $(-2x + 5)(3x - 2) = -6x^2 + (15 + 4)x - 10 = -6x^2 + 19x - 10$
6. $(-5x - 3)(-2x + 4) = 10x^2 + (6 - 20)x - 12 = 10x^2 - 14x - 12$

1. $5x(3x + 2y) - 3x(2x - 4y) = (15 - 6)x^2 + (10 + 12)xy = 9x^2 + 22xy$
2. $(2x - 3y)(3x + 2y) + (5x - y)(2x + 3y) = (6x^2 - 5xy - 6y^2) + (10x^2 + 13xy - 3y^2) = 16x^2 + 8xy - 9y^2$
3. $(3a - 2b)(2a + b) - (2a + 3b)(3a - b) = (6a^2 - ab - 2b^2) - (6a^2 + 7ab - 3b^2) = b^2 - 8ab = b(b - 8a)$
4. $(2a + 3)(3a - 2)(2a - 5) = (6a^2 + 5a - 6)(2a - 5) = 12a^3 - 20a^2 - 37a + 30$
5. $(2x + 3)(2x - 5)(3x - 4) = (4x^2 - 4x - 15)(3x - 4) = 12x^3 - 28x^2 - 29x + 60$

1. $P(x) \times Q(x) + P(x) \times S(x) = (3x^2)(2x + 1) + (3x^2)(5x + 4) = 21x^3 + 15x^2$
2. $P(x) \times Q(x) - Q(x) \times S(x) = (3x^2)(2x + 1) - (3x^2)(5x + 4) = -9x^3 - 9x^2$
3. $P(x) \times Q(x) \times S(x) = (3x^2)(2x + 1)(5x + 4) = (6x^3 + 3x^2)(5x + 4) = 30x^4 + 39x^3 + 12x^2$
4. $P(x) \times (Q(x) + S(x)) = (3x^2)((2x + 1) + (5x + 4)) = (3x^2)(7x + 5) = 21x^3 + 15x^2$
5. $(P(x) - Q(x)) \times S(x) = ((3x^2) - (2x + 1))(5x + 4) = (3x^2 - 2x - 1)(5x + 4) = 15x^3 + 2x^2 - 13x - 4$
6. $(S(x) - Q(x)) \times P(x) = ((5x + 4) - (2x + 1))(3x^2) = (3x + 3)(3x^2) = 9x^3 + 9x^2$

1. $(x + 5)^2 = x^2 + 10x + 25$
2. $(3x + 4)^2 = 9x^2 + 24x + 16$
3. $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$
4. $(2x^3 + 5x^2)^2 = 4x^6 + 20x^5 + 25x^4$
5. $(3x^2y + xy^2)^2 = 9x^4y^2 + 6x^3y^3 + x^2y^4$
6. $(7x^2y + 3x^5y^2)^2 = 49x^4y^2 + 42x^7y^3 + 9x^{10}y^4$
7. $\left(\frac{1}{2}x + 7\right)^2 = \frac{1}{4}x^2 + 7x + 49$
8. $\left(\frac{3}{4}x^2 + \frac{2}{9}y^2\right)^2 = \frac{9}{16}x^4 + \frac{1}{3}x^2y^2 + \frac{4}{81}y^4$

1. $(x - 4)^2 = x^2 - 8x + 16$
2. $(2x - 7)^2 = 4x^2 - 28x + 49$
3. $(4x - y)^2 = 16x^2 - 8xy + y^2$
4. $(2x^2 - 3x)^2 = 4x^4 - 12x^3 + 9x^2$
5. $(-3x - 4y)^2 = 9x^2 + 24xy + 16y^2$
6. $(4x^2 - 5y)^2 = 16x^4 - 40x^2y + 25y^2$

7. $\left(\frac{3}{4}x - \frac{1}{2}\right)^2 = \frac{9}{16}x^2 - \frac{3}{4}x + \frac{1}{4}$

8. $\left(\frac{5}{6}x - 3y^2\right)^2 = \frac{25}{36}x^2 - 5xy^2 + 9y^4$

Exercice 48

1. $(x + 5)(x - 5) = x^2 - 25$

2. $(2x - 7)(2x + 7) = 4x^2 - 49$

3. $(4x + 3y)(4x - 3y) = 16x^2 - 9y^2$

4. $(2 - 3x)(2 + 3x) = 4 - 9x^2$

5. $(-2x + 3y)(-2x - 3y) = 4x^2 - 9y^2$

6. $(3x^2 + 5)(3x^2 - 5) = 9x^4 - 25$

7. $\left(\frac{2}{3}x + \frac{4}{5}\right)\left(\frac{2}{3}x - \frac{4}{5}\right) = \frac{4}{9}x^2 - \frac{16}{25}$

8. $\left(\frac{x}{6} - \frac{3}{4}y^2\right)\left(\frac{x}{6} + \frac{3}{4}y^2\right) = \frac{x^2}{36} - \frac{9}{16}y^4$

Exercice 49

1. $(3x + 2)^2 + (3x - 2)^2 = (9x^2 + 6x + 4) + (9x^2 - 6x + 4) = 18x^2 + 8$

2. $(2x^2 + 5)^2 - (2x^2 - 5)^2 = (4x^4 + 20x^2 + 25) - (4x^4 - 20x^2 + 25) = 40x^2$

3. $(x + 1)(x^2 + 1)(x - 1) = (x^3 + x^2 + x + 1)(x - 1) = x^4 - 1$

4. $3x(5x - 4)^2 - 2x(3x + 5)^2 = 3x(25x^2 - 40x + 16) - 2x(9x^2 + 30x + 25) = 57x^3 - 180x^2 - 2x$

Exercice 50

1. $(24x^4 + 12x^3 - 18x^2) \div 6x^2 = 4x^2 + 2x - 3$

2. $(36x^2y^4 + 27x^3y^2 - 9x^2y^2) \div 9x^2y = 4y^3 + 3xy - y$

3. $(3x^2 + 2x)(4x + 6) \div 2x = \frac{(3x^2 + 2x)\,(4x + 6)}{2} = (3x + 2)(2x + 3) = 6x^2 + 13x + 6$

4. $(3x + 6)^2 \div 3x = \frac{9x^2 + 36x + 36}{3x} = 3x + 12 + \frac{12}{x}$

5. $(4x^2 + 2x)(4x^2 - 2x) \div 4x^2 = \frac{16x^4 - 4x^2}{4x^2} = 4x^2 - 1$

Exercice 51 Comme $2x^2 + 5x - 3 = (x - 1)(2x + 7) + 4$, le quotient $Q(x) = 2x + 7$ et le reste $R(x) = 4$ de la division de $A(x)$ par $B(x)$ vérifient la relation euclidienne $A(x) = B(x) \times Q(x) + R(x)$.

Exercice 52

1. On a $2x^2 - x - 6 = (2x + 3)(x - 2)$ donc $Q(x) = x - 2$ et $R(x) = 0$.

2. On a $3x^2 - 2x + 1 = (x + 1)(3x - 5) + 6$ donc $Q(x) = 3x - 5$ et $R(x) = 6$.

3. On a $2x^3 + 3x^2 + 2x + 4 = (x + 1)(2x^2 + x + 1) + 3$ donc $Q(x) = 2x^2 + x + 1$ et $R(x) = 3$.

4. On a $x^3 - 2x + 1 = (x - 1)(x^2 + x - 1)$ donc $Q(x) = x^2 + x + 1$ et $R(x) = 0$.

5. On a $x^4 - 1 = (x + 1)(x^3 - x^2 + x - 1)$ donc $Q(x) = x^3 - x^2 + x - 1$ et $R(x) = 0$.

6. On a $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$ donc $Q(x) = x^2 - 3x + 9$ et $R(x) = 0$.

Exercice 53

1. (a) $P(2) = 3(2)^2 - 5(2) + 1 = 3$.

   (b) Comme $P(x) = (x - 2)(3x + 1) + 3$, le reste de la division de $P(x)$ par $(x - 2)$ est bien égal à $P(2) = 3$.

2. (a) $P(-2) = 3(-2)^2 - 5(-2) + 1 = 23$.

   (b) Comme $P(x) = (x + 2)(3x - 11) + 23$, le reste de la division de $P(x)$ par $(x + 2)$ est égal à $P(-2) = 23$.

Exercice 54

1. On montre aisément que $P(x) = x^3 + 2x^2 - 5x - 6 = (x + 3)(x - 2)(x + 1)$, ce qui pouve que $P(x)$ est divisible par $x + 3$, $x - 2$ et $x + 1$.

2. (a) On a $P(x) = (x - 1)(x^2 + 3x - 2) - 8$ donc $P(x)$ n'est pas divisible par $x - 1$ (reste non nul).

   (b) On a $P(x) = (x + 2)(x^2 - 5) + 4$ donc $P(x)$ n'est pas divisible par $x + 2$ (reste non nul).

   (c) On a $P(x) = (x - 3)(x^2 + 5x + 10) + 24$ donc $P(x)$ n'est pas divisible par $x - 3$ (reste non nul).

Exercice 55

1. On a $P(2) = 2(2)^3 + 3(2)^2 - 4(2) - 1 = 19$ donc le reste de la division de $P(x)$ par $x - 2$ est $R(x) = 19$.
2. On a $P(-2) = 2(-2)^3 + 3(-2)^2 - 4(-2) - 1 = 3$ donc le reste de la division de $P(x)$ par $x + 2$ est $R(x) = 3$.
3. On a $P(1) = 2(1)^3 + 3(1)^2 - 4(1) - 1 = 0$ donc le reste de la division de $P(x)$ par $x - 1$ est $R(x) = 0$.

Exercice 56

1. Le plus grand facteur commun parmi $18x^4$, $24x^3$, $12x^5$ est $6x^3$.
2. Le plus grand facteur commun parmi $18x^3y^2z^4$, $24x^4y^3z^4$, $36x^2y^4z^3$ est $6x^2y^2z^3$.
3. Le plus grand facteur commun parmi $15x^2(a+b)^3$, $18x^3(a+b)^2$ est $3x^2(a+b)^2$.
4. Le plus grand facteur commun parmi $24x^3y^2(a-b)^3$, $26x^2y^4(a-b)^2$ est $2x^2y^2(a-b)^2$.

Exercice 57

1. $5x - 10 = 5(x - 2)$
2. $18x + 24y - 12z = 6(3x + 4y - 2z)$
3. $4x^2 + 6x = 2x(2x + 3)$
4. $12x + x^2 - 5x^3 = x(12 + x - 5x^2)$
5. $12a^2b + 18a^2b^2 = 6a^2b(2 + 3b)$
6. $-3x^4 + 6x^3 - 9x^2 = 3x^2(-x^2 + 2x - 3)$
7. $a^2 + ab + a = a(a + b + 1)$
8. $x^4 - x^3y - x^2 = x^2(x^2 - xy - 1)$
9. $24x^3y^2 - 16x^2y^3 + 28x^3y^4 = 4x^2y^2(6x - 4y + 7xy^2)$
10. $21x^3y^2z - 14x^2y^3z^2 + 28x^2y^2z^2 = 7x^2y^2z(3x - 2yz + 4z)$

Exercice 58

1. $x(x + 2) + 5(x + 2) = (x + 5)(x + 2)$
2. $3(x - 2) - x(x - 2) = (3 - x)(x - 2)$
3. $a(b + c) - d(b + c) = (a - d)(b + c)$
4. $x(3 - y) + y(3 - y) = (x + y)(3 - y)$
5. $(x + 3)(x + 2) + (x + 3)(x - 1) = (x + 3)((x + 2) + (x - 1)) = (x + 3)(2x + 1)$
6. $(x + y)(x - 2) - (x + y)(2x - 3) = (x + y)(-x + 1)$
7. $(x + y)^2 + x(x + y) = (2x + y)(x + y)$
8. $(x - y)^2 + (x - y)(x + y) = (x - y)(2x)$
9. $2(x - 1) - (x - 1)^2 = (-x + 3)(x - 1)$
10. $(2x - 3)^2 + (x + 1)(2x - 3) + (2x - 3)(x + 3) = (2x - 3)((2x - 3) + (x + 1) + (x + 3)) = (2x - 3)(4x + 1)$

Exercice 59

1. $x(x - 1) - 3(1 - x) = (x + 3)(x - 1)$
2. $x(x + 3) + 2(-x - 3) = (x - 2)(x + 3)$
3. $(x - 5)^2 - 2(5 - x) = ((x - 5) + 2)(x - 5) = (x - 3)(x - 5)$
4. $(2x + 1)(2x - 1) + (1 - 2x)^2 = ((2x + 1) - (1 - 2x))(2x - 1) = (4x)(2x - 1)$
5. $(2x + 3y)(x + y) + (4x + 6y)(x - y) = (2x + 3y)((x + y) + 2(x - y)) = (2x + 3y)(3x - y)$
6. $(x + 1)(2x + 6) - (x - 2)(3x + 9) = 2(x + 1)(x + 3) - 3(x - 2)(x + 3) = (-x + 8)(x + 3)$

Exercice 60

1. $(3a + 2b)^2 - (3a + 2b)(2a - 3b) + (3a + 2b) = (3a + 2b)((3a + 2b) - (2a - 3b) + 1) = (3a + 2b)(a + 5b + 1)$
2. $(x + 5)^2 + x^2 + 5x = (x + 5)^2 + x(x + 5) = (2x + 5)(x + 5)$
3. $(4x + 7)^2 + (5 - x)(4x + 7) + 4x^2 + 7x = ((4x + 7) + (5 - x) + x)(4x + 7) = (4x + 12)(4x + 7)$
4. $(3x + 2)^2 + (3x + 2)(2x - 3) - (3x + 2) = (3x + 2)((3x + 2) + (2x - 3) - 1) = (3x + 2)(5x - 2)$
5. $(3x - 4)(x + 1) + 6x^2 - 8x + (3x - 4)(x - 3) = (3x - 4)((x + 1) + 2x + (x - 3)) = (3x - 4)(4x - 2)$

1. $x^2 + 5xy + 3x + 15y = x(x + 3) + 5y(x + 3) = (x + 5y)(x + 3)$
2. $2x^2 + 3xy - 10x - 15y = 2x(x - 5) + 3y(x - 5) = (2x + 3y)(x - 5)$
3. $6a^2 - 15a + 2ab - 5b = 3a(2a - 5) + b(2a - 5) = (3a + b)(2a - 5)$
4. $6x^2 - 8x - 9xy + 12y = 2x(3x - 4) - 3y(3x - 4) = (2x - 3y)(3x - 4)$
5. $10xy + 2x + 15y + 3 = 2x(5y + 1) + 3(5y + 1) = (2x + 3)(5y + 1)$
6. $x^3 - x^2 + x - 1 = (x - 1)(x^2 + 1)$

1. $2x^2y + 3x^2 + 10y + 15 = x^2(2y + 3) + 5(2y + 3) = (x^2 + 5)(2y + 3)$
2. $15x^4y^2 + 35x^2y^2 - 9x^2 - 21 = 5x^2y^2(3x^2 + 7) - 3(3x^2 + 7) = (5x^2y^2 - 3)(3x^2 + 7)$
3. $2x^3 + 4x^2y - 2x^2 - 4xy = x(2x^2 + 4xy) - (2x^2 + 4xy) = (x - 1)(2x^2 + 4xy) = 2x(x - 1)(x + 2y)$
4. $3x^3y - 9x^3 + 6x^2y - 18x^2 = x(3x^2y - 9x^2) + 2(3x^2y - 9x^2) = (x + 2)(3x^2y - 9x^2) = 3x^2(x + 2)(y - 3)$
5. $30x^4y - 10x^3y^2 + 15x^3y - 5x^2y^2 = 10x(3x^3y - x^2y^2) + 5(3x^3y - x^2y^2) = (10x + 5)(3x^3y - x^2y^2) = 5x^2y(2x + 1)(3x - y)$
6. $2x^4 - 2x^3 + 6x^2 - 6x = 2x^2(x^2 - x) + 6(x^2 - x) = (2x^2 + 6)(x^2 - x) = 2x(x^2 + 3)(x - 1)$

1. $ax - ay + bx - by + cx - cy = a(x - y) + b(x - y) + c(x - y) = (a + b + c)(x - y)$
2. $6ax - 3ay + 10bx - 5by - 4x + 2y = 3a(2x - y) + 5b(2x - y) - 2(2x - y) = (3a + 5b - 2)(x - y)$
3. $a^3 - 2ab + ac^2 - a^2b + 2b^2 - bc^2 + a^2c - 2bc + c^3 = a^2(a - b + c) - 2b(a - b + c) + c^2(a - b + c) = (a^2 - 2b + c^2)(a - b + c)$
4. $ab(x^2 + y^2) - xy(a^2 + b^2) = ax(bx - ay) + by(ay - bx) = (ax - by)(bx - ay)$

1. $x^2 - 25 = (x - 5)(x + 5)$
2. $16x^2 - 9 = (4x - 3)(4x + 3)$
3. $49x^2 - 36y^2 = (7x - 6y)(7x + 6y)$
4. $36x^4 - 25y^6 = (6x^2 - 5y^3)(6x^2 + 5y^3)$
5. $100 - x^2 = (10 - x)(10 + x)$
6. $\frac{x^2}{16} - \frac{y^2}{9} = \left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x}{4} + \frac{y}{3}\right)$
7. $x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$
8. $x^2 - 1 = (x - 1)(x + 1)$
9. $16x^2 - \frac{1}{9} = (4x - \frac{1}{3})(4x + \frac{1}{3})$
10. $\frac{25}{16}x^2y^4 - \frac{4}{9}z^6 = \left(\frac{5}{4}xy^2 - \frac{2}{3}z^3\right)\left(\frac{5}{4}xy^2 + \frac{2}{3}z^3\right)$

1. $(3x - 1)^2 - 9 = ((3x - 1) - 3)((3x - 1) + 3) = (3x - 4)(3x + 2)$
2. $(x + 1)^2 - 4 = ((x + 1) - 2)((x + 1) + 2) = (x - 1)(x + 3)$
3. $(2x + 5)^2 - 16x^2 = ((2x + 5) - 4x)((2x + 5) + 4x) = (-2x + 5)(6x + 5)$
4. $25x^2 - (2x - 5)^2 = (5x - (2x - 5))(5x + (2x - 5)) = (3x + 5)(7x - 5)$
5. $16x^2 - (3x + 2)^2 = (4x - (3x + 2))(4x + (3x + 2)) = (x - 2)(7x + 2)$
6. $36x^2 - (2 - x)^2 = (6x - (2 - x))(6x + (2 - x)) = (7x - 2)(5x + 2)$
7. $(x + 3)^2 - (2x + 5)^2 = ((x + 3) - (2x + 5))((x + 3) + (2x + 5)) = (-x - 2)(3x + 8)$
8. $(3x - 5y)^2 - (2x - 3y)^2 = ((3x - 5y) - (2x - 3y))((3x - 5y) + (2x - 3y)) = (x - 2y)(5x - 8y)$
9. $4(x + 5)^2 - 1 = (2(x + 5) - 1)(2(x + 5) + 1) = (2x + 9)(2x + 11)$
10. $16x^2 - (3x + 2)^2 = (4x - (3x + 2))(4x + (3x + 2)) = (x - 2)(7x + 2)$
11. $25(x - 3)^2 - 9(2x + 1)^2 = (5(x - 3) - 3(2x + 1))(5(x - 3) + 3(2x + 1)) = (-x - 18)(11x - 12)$

$\boxed{\text{Exercice 66}}$

1. $2x^3 - 18x = 2x(x^2 - 9) = 2x(x-3)(x+3)$

2. $2x^3 + 3x^2 - 2xy^2 - 3y^2 = x^2(2x+3) - y^2(2x+3) = (x^2 - y^2)(2x+3)$

3. $25x^3 - 50x^2 - 9xy^2 + 18y^2 = 25x^2(x-2) - 9y^2(x-2) = (25x^2 - 9y^2)(x-2) = (5x-3y)(5x+3y)(x-2)$

4. $x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x-3)(x+3)(x^2+9)$

5. $x^2 - 1 + (x-1)^2 = (x-1)(x+1) + (x-1)^2 = (x-1)((x+1) + (x-1)) = 2x(x-1)$

6. $2x^2 - \frac{1}{2} = 2(x^2 - \frac{1}{4}) = 2(x - \frac{1}{2})(x + \frac{1}{2})$

$\boxed{\text{Exercice 67}}$

1. $x^2 + 10x + 25 = (x+5)^2$

2. $x^2 - 14x + 49 = (x-7)^2$

3. $4x^2 + 12xy + 9y^2 = (2x+3y)^2$

4. $25x^2 - 20xy + 4y^2 = (5x-2y)^2$

5. $9x^4 - 30x^2 + 25 = (3x^2 - 5)^2$

6. $25x^4 + 30x^2y^3 + 9y^6 = (5x^2 + 3y^3)^2$

7. $x^2 - x + \frac{1}{4} = (x - \frac{1}{2})^2$

8. $\frac{9}{16}x^2 + x + \frac{4}{9} = (\frac{3}{4}x + \frac{2}{3})^2$

$\boxed{\text{Exercice 68}}$

1. On a $\Delta = (6)^2 - 4(4)(9) = -108 < 0$ donc le trinôme n'est pas factorisable (dans $\mathbb{R}$) et n'est donc pas un carré parfait.

2. On a $\Delta = (12)^2 - 4(4)(-9) = 288 > 0$ donc, d'après la méthode de complétion,

$$4x^2 + 12x - 9 = 4(x^2 + 3x - \tfrac{9}{4}) = 4((x + \tfrac{3}{2})^2 - \tfrac{9}{2}) = 4(x + \tfrac{3}{2} + \tfrac{3\sqrt{2}}{2})(x + \tfrac{3}{2} - \tfrac{3\sqrt{2}}{2})$$

et le trinôme n'est pas un carré parfait.

3. On a $\Delta = (12)^2 - 4(-4)(9) = 288 > 0$ donc, d'après la méthode de complétion,

$$-4x^2 + 12x + 9 = -4(x^2 - 3x - \tfrac{9}{4}) = 4((x - \tfrac{3}{2})^2 - \tfrac{9}{2}) = 4(x - \tfrac{3}{2} - \tfrac{3\sqrt{2}}{2})(x - \tfrac{3}{2} + \tfrac{3\sqrt{2}}{2})$$

et le trinôme n'est pas un carré parfait.

4. On a $\Delta = (15)^2 - 4(9)(25) = -675 < 0$ donc le trinôme n'est pas factorisable (dans $\mathbb{R}$) et n'est donc pas un carré parfait.

$\boxed{\text{Exercice 69}}$

1. $x^2 + \ldots + 9 = x^2 + 6x + 9 = (x+3)^2$

2. $4x^2 - \ldots + 9 = 4x^2 - 12x + 9 = (2x-3)^2$

3. $9x^2 + 30x + \ldots = 9x^2 + 30x + 25 = (3x+5)^2$

4. $\ldots + 20x + 4 = 25x^2 + 20x + 4 = (5x+2)^2$

5. $4x^2 - 28x + \ldots = 4x^2 - 28x + 49 = (2x-7)^2$

6. $\ldots - 6x + 1 = 9x^2 - 6x + 1 = (3x-1)^2$

7. $x^2 + \frac{2}{3}x + \ldots = x^2 + \frac{2}{3}x + \frac{1}{9} = (x + \frac{1}{3})^2$

8. $x^2 - \ldots + \frac{49}{4} = x^2 - 7x + \frac{49}{4} = (x - \frac{7}{2})^2$

$\boxed{\text{Exercice 70}}$

1. $4x^3 - 12x^2 + 9x = x(4x^2 - 12x + 9) = x(2x-3)^2$

2. $x^4 - 2x^2 + 1 = (x^2 - 1)^2 = ((x-1)(x+1))^2 = (x-1)^2(x+1)^2$

3. $-x^2 + 6x - 9 = -(x^2 - 6x + 9) = -(x-3)^2$

4. $(x-1)(2x+1) + x^2 - 2x + 1 = (x-1)(2x+1) + (x-1)^2 = (x-1)[(2x+1) + (x-1)] = 3x(x-1)$

5. $(x^2+4)^2 - 16x^2 = (x^2+4)^2 - (4x)^2 = [(x^2+4) - 4x][(x^2+4) + 4x] = (x-2)^2(x+2)^2$

6. $x^2(x+1) + 2x(x+1) + (x+1) = (x^2 + 2x + 1)(x+1) = (x+1)^2(x+1) = (x+1)^3$

1. On a $a = 2$, $b = 3$, $c = 1$ donc $\Delta = (3)^2 - 4(2)(1) = 1 > 0$. Par conséquent, le trinôme est factorisable et
$$2x^2 + 3x + 1 = 2(x^2 + \tfrac{3}{2}x + \tfrac{1}{2}) = 2[(x + \tfrac{3}{4})^2 - \tfrac{1}{16}] = 2[(x + \tfrac{3}{4} - \tfrac{1}{4})(x + \tfrac{3}{4} + \tfrac{1}{4})] = 2(x + \tfrac{1}{2})(x + 1)$$

2. On a $a = 2$, $b = -1$, $c = -6$ donc $\Delta = (-1)^2 - 4(2)(-6) = 25 > 0$. Par conséquent, le trinôme est factorisable et
$$2x^2 - x - 6 = 2(x^2 - \tfrac{x}{2} - 3) = 2[(x - \tfrac{1}{4})^2 - \tfrac{49}{16}] = 2[(x - \tfrac{1}{4} - \tfrac{7}{4})(x - \tfrac{1}{4} + \tfrac{7}{4})] = 2(x - 2)(x + \tfrac{3}{2})$$

3. On a $a = 8$, $b = 2$, $c = -15$ donc $\Delta = (2)^2 - 4(8)(-15) = 484 > 0$. Par conséquent, le trinôme est factorisable et
$$8x^2 + 2x - 15 = 8(x^2 + \tfrac{1}{4}x - \tfrac{15}{8}) = 8[(x + \tfrac{1}{8})^2 - \tfrac{121}{64}] = 8(x + \tfrac{1}{8} - \tfrac{11}{8})(x + \tfrac{1}{8} + \tfrac{11}{8}) = 8(x - \tfrac{5}{4})(x + \tfrac{6}{4})$$

4. On a $a = -3$, $b = 5$, $c = 2$ donc $\Delta = (5)^2 - 4(-3)(2) = 49 > 0$. Par conséquent, le trinôme est factorisable et
$$-3x^2 + 5x + 2 = -3(x^2 - \tfrac{5}{3}x - \tfrac{2}{3}) = -3[(x - \tfrac{5}{6})^2 - \tfrac{49}{36}] = -3(x - \tfrac{5}{6} - \tfrac{7}{6})(x - \tfrac{5}{6} + \tfrac{7}{6}) = -3(x - 2)(x + \tfrac{1}{3})$$

5. On a $a = 1$, $b = -1$, $c = 1$ donc $\Delta = (-1)^2 - 4(1)(1) = -3 < 0$ donc le trinôme n'est pas factorisable (dans $\mathbb{R}$).

6. On a $a = 4$, $b = -12$, $c = 9$ donc $\Delta = (-12)^2 - 4(4)(9) = 0$. Par conséquent, le trinôme $4x^2 - 12x + 9$ est factorisable et $4x^2 - 12x + 9 = (2x - 3)^2$.

1. $x^2 - 10x + 21 = (x - 5)^2 - 4 = (x - 5 - 2)(x - 5 + 2) = (x - 7)(x - 3)$
2. $x^2 - 5x - 14 = (x - \tfrac{5}{2})^2 - \tfrac{81}{4} = (x - \tfrac{5}{2} - \tfrac{9}{2})(x - \tfrac{5}{2} + \tfrac{9}{2}) = (x - 7)(x + 2)$
3. $x^2 - 7x + 12 = (x - \tfrac{7}{2})^2 - \tfrac{1}{4} = (x - \tfrac{7}{2} - \tfrac{1}{2})(x - \tfrac{7}{2} + \tfrac{1}{2}) = (x - 4)(x - 3)$
4. $x^2 - 9x + 20 = (x - \tfrac{9}{2})^2 - \tfrac{1}{4} = (x - \tfrac{9}{2} - \tfrac{1}{2})(x - \tfrac{9}{2} + \tfrac{1}{2}) = (x - 5)(x - 4)$
5. $2x^2 + 7x + 3 = 2(x^2 + \tfrac{7}{2}x + \tfrac{3}{2}) = 2[(x + \tfrac{7}{4})^2 - \tfrac{25}{16}] = 2(x + \tfrac{7}{4} - \tfrac{5}{4})(x + \tfrac{7}{4} + \tfrac{5}{4}) = 2(x + \tfrac{1}{2})(x + 3)$
6. $3x^2 + 5x - 2 = 3(x^2 + \tfrac{5}{3}x - \tfrac{2}{3}) = 3[(x + \tfrac{5}{6})^2 - \tfrac{49}{36}] = 3(x + \tfrac{5}{6} - \tfrac{7}{6})(x + \tfrac{5}{6} + \tfrac{7}{6}) = 3(x - \tfrac{1}{3})(x + 2)$
7. $6x^2 + x - 2 = 6(x^2 + \tfrac{x}{6} - \tfrac{1}{3}) = 6[(x + \tfrac{1}{12})^2 - \tfrac{49}{144}] = 6(x + \tfrac{1}{12} - \tfrac{7}{12})(x + \tfrac{1}{12} + \tfrac{7}{12}) = 6(x - \tfrac{1}{2})(x + \tfrac{2}{3})$
8. $10x^2 - 19x + 6 = 10(x^2 - \tfrac{19}{10}x + \tfrac{6}{10}) = 10[(x - \tfrac{19}{20})^2 - \tfrac{121}{400}] = 10(x - \tfrac{19}{20} - \tfrac{11}{20})(x - \tfrac{19}{20} + \tfrac{11}{20}) = 10(x - \tfrac{3}{2})(x - \tfrac{2}{5})$

1. $x^2 + 8x + 15 = (x + 4)^2 - 1 = (x + 4 - 1)(x + 4 + 1) = (x + 3)(x + 5)$
2. $x^2 - 8x + 15 = (x - 4)^2 - 1 = (x - 4 - 1)(x - 4 + 1) = (x - 5)(x - 3)$
3. $x^2 + 5x - 14 = (x + \tfrac{5}{2})^2 - \tfrac{81}{4} = (x + \tfrac{5}{2} - \tfrac{9}{2})(x + \tfrac{5}{2} + \tfrac{9}{2}) = (x - 2)(x + 7)$
4. $x^2 - 5x - 14 = (x - \tfrac{5}{2})^2 - \tfrac{81}{4} = (x - \tfrac{5}{2} - \tfrac{9}{2})(x - \tfrac{5}{2} + \tfrac{9}{2}) = (x - 7)(x + 2)$
5. $6x^2 + 19x + 15 = 6(x^2 + \tfrac{19}{6}x + \tfrac{15}{6}) = 6[(x + \tfrac{19}{12})^2 - \tfrac{1}{144}] = 6(x + \tfrac{19}{12} - \tfrac{1}{12})(x + \tfrac{19}{12} + \tfrac{1}{12}) = 6(x + \tfrac{3}{2})(x + \tfrac{5}{3})$
6. $2x^2 - 7x - 15 = 2(x^2 - \tfrac{7}{2}x - \tfrac{15}{2}) = 2[(x - \tfrac{7}{4})^2 - \tfrac{169}{16}] = 2(x - \tfrac{7}{4} - \tfrac{13}{4})(x - \tfrac{7}{4} + \tfrac{13}{4}) = 2(x - 5)(x + \tfrac{3}{2})$
7. $3x^2 - x - 4 = 3(x^2 - \tfrac{x}{3} - \tfrac{4}{3}) = 3[(x - \tfrac{1}{6})^2 - \tfrac{49}{36}] = 3(x - \tfrac{1}{6} - \tfrac{7}{6})(x - \tfrac{1}{6} + \tfrac{7}{6}) = 3(x - \tfrac{4}{3})(x + 1)$
8. $5x^2 - 17x + 6 = 5(x^2 - \tfrac{17}{5}x + \tfrac{6}{5}) = 5[(x - \tfrac{17}{10})^2 - \tfrac{169}{100}] = 5(x - \tfrac{17}{10} - \tfrac{13}{10})(x - \tfrac{17}{10} + \tfrac{13}{10}) = 5(x - 3)(x - \tfrac{2}{5})$

1. $4x^2 - 12x + 9 = (2x - 3)^2$
2. $x^2 + 6xy + 8y^2 = (x + 3y)^2 - y^2 = (x + 3y - y)(x + 3y + y) = (x + 2y)(x + 4y)$
3. $x^2 - 2x - 1 = (x - 1)^2 - 2 = (x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$
4. $x^2 + 6x + 7 = (x + 3)^2 - 2 = (x + 3 - \sqrt{2})(x + 3 + \sqrt{2})$

1. On a $a = 2, b = 9, c = 4$. On recherche ensuite m, n tels que
$$\begin{cases} mn = 8 \\ m + n = 9 \end{cases} \Leftrightarrow \begin{cases} m = 1 \\ n = 8 \end{cases}$$
et on écrit $2x^2 + 9x + 4 = 2x^2 + x + 8x + 4 = 2x(x + \tfrac{1}{2}) + 8(x + \tfrac{1}{2}) = (2x + 8)(x + \tfrac{1}{2})$.

2. On a $a = 6, b = -19, c = 10$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & 60 \\ m+n & = & -19 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & -4 \\ n & = & -15 \end{array}\right.$$

et on écrit $6x^2 - 19x + 10 = 6x^2 - 4x - 15x + 10 = 2x(3x-2) - 5(3x-2) = (2x-5)(3x-2)$.

3. On a $a = 4, b = -5, c = -21$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & -84 \\ m+n & = & -5 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & -12 \\ n & = & 7 \end{array}\right.$$

et on écrit $4x^2 - 5x - 21 = 4x^2 - 12x + 7x - 21 = 4x(x-3) + 7(x-3) = (4x+7)(x-3)$.

4. On a $a = 5, b = -32, c = -21$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & -105 \\ m+n & = & -32 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & -35 \\ n & = & 3 \end{array}\right.$$

et on écrit $5x^2 - 32x - 21 = 5x^2 - 35x + 3x - 21 = 5x(x-7) + 3(x-7) = (5x+3)(x-7)$.

5. On a $a = 12, b = 13, c = 3$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & 36 \\ m+n & = & 13 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & 9 \\ n & = & 4 \end{array}\right.$$

et on écrit $12x^2 + 13x + 3 = 12x^2 + 9x + 4x + 3 = 3x(4x+3) + (4x+3) = (3x+1)(4x+3)$.

6. On a $a = 16, b = -26, c = 3$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & 48 \\ m+n & = & -26 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & -24 \\ n & = & -2 \end{array}\right.$$

et on écrit $16x^2 - 26x + 3 = 16x^2 - 24x - 2x + 3 = 8x(2x-3) - (2x-3) = (8x-1)(2x-3)$.

7. On a $a = 6, b = 11, c = -10$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & -60 \\ m+n & = & 11 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & 15 \\ n & = & -4 \end{array}\right.$$

et on écrit $6x^2 + 11x - 10 = 6x^2 + 15x - 4x - 10 = 3x(2x+5) - 2(2x+5) = (3x-2)(2x+5)$.

8. On a $a = 8, b = 2, c = -15$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & -120 \\ m+n & = & 2 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & 12 \\ n & = & -10 \end{array}\right.$$

et on écrit $8x^2 + 2x - 15 = 8x^2 + 12x - 10x - 15 = 4x(2x+3) - 5(2x+3) = (4x-5)(2x+3)$.

9. On a $a = 1, b = 10, c = 24$. On recherche ensuite m, n tels que

$$\left\{\begin{array}{rcl} mn & = & 24 \\ m+n & = & 10 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & 6 \\ n & = & 4 \end{array}\right.$$

et on écrit $x^2 + 10x + 24 = x^2 + 6x + 4x + 24 = x(x+6) + 4(x+6) = (x+4)(x+6)$.

10. On a $a = 1, b = -7, c = 12$. On recherche ensuite mn, tels que

$$\left\{\begin{array}{rcl} mn & = & 12 \\ m+n & = & -7 \end{array}\right. \Leftrightarrow \left\{\begin{array}{rcl} m & = & -4 \\ n & = & -3 \end{array}\right.$$

et on écrit $x^2 - 7x + 12 = x^2 - 4x - 3x + 12 = x(x-4) - 3(x-4) = (x-3)(x-4)$.

Exercice 76   On corrige cet exercice même s'il est pour l'instant hors sujet...

1. On a $\Delta = (9)^2 - 4(2)(4) = 49 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(9)-\sqrt{49}}{2(2)} = -4$ et $x_2 = \frac{-(9)+\sqrt{49}}{2(2)} = -\frac{1}{2}$ donc le trinôme $2x^2 + 9x + 4$ se réécrit sous la forme $2(x+4)(x+\frac{1}{2}) = (2x+8)(x+\frac{1}{2})$.

2. On a $\Delta = (-19)^2 - 4(6)(10) = 121 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(-19)-\sqrt{121}}{2(6)} = \frac{2}{3}$ et $x_2 = \frac{-(-19)+\sqrt{121}}{2(6)} = \frac{5}{2}$ donc le trinôme $6x^2 - 19x + 10$ se réécrit sous la forme $6(x - \frac{5}{2})(x - \frac{2}{3}) = (2x-5)(3x-2)$.

3. On a $\Delta = (-5)^2 - 4(4)(-21) = 361 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(-5)-\sqrt{361}}{2(4)} = -\frac{7}{4}$ et $x_2 = \frac{-(-5)+\sqrt{361}}{2(4)} = 3$ donc le trinôme $4x^2 - 5x - 21$ se réécrit $4(x + \frac{7}{4})(x - 3) = (4x+7)(x-3)$.

4. On a $\Delta = (-32)^2 - 4(5)(-21) = 1444 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(-32)-\sqrt{1444}}{2(5)} = -\frac{3}{5}$ et $x_2 = \frac{-(-32)+\sqrt{1444}}{2(5)} = 7$ donc le trinôme $5x^2 - 32x - 21$ se réécrit $5(x+\frac{3}{5})(x-7) = (5x+3)(x-7)$.

5. On a $\Delta = (13)^2 - 4(12)(3) = 25 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(13)-\sqrt{25}}{2(12)} = -\frac{3}{4}$ et $x_2 = \frac{-(13)+\sqrt{25}}{2(12)} = -\frac{1}{3}$ donc le trinôme $12x^2 + 13x + 3$ se réécrit $12(x + \frac{3}{4})(x + \frac{1}{3}) = (4x + 3)(3x + 1)$.

6. On a $\Delta = (-26)^2 - 4(16)(3) = 484 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(-26)-\sqrt{484}}{2(16)} = \frac{1}{8}$ et $x_2 = \frac{-(-26)+\sqrt{484}}{2(16)} = \frac{3}{2}$ donc le trinôme $16x^2 - 26x + 3$ se réécrit $16(x - \frac{1}{8})(x - \frac{3}{2}) = (8x - 1)(2x - 3)$.

7. On a $\Delta = (11)^2 - 4(6)(-10) = 361 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(11)-\sqrt{361}}{2(6)} = -\frac{5}{2}$ et $x_2 = \frac{-(11)+\sqrt{361}}{2(6)} = \frac{2}{3}$ donc le trinôme $6x^2 + 11x - 10$ se réécrit $6(x + \frac{5}{2})(x - \frac{2}{3}) = (2x + 5)(3x - 2)$.

8. On a $\Delta = (2)^2 - 4(8)(-15) = 484 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(2)-\sqrt{484}}{2(8)} = -\frac{3}{2}$ et $x_2 = \frac{-(2)+\sqrt{484}}{2(8)} = \frac{5}{4}$ donc le trinôme $8x^2 + 2x - 15$ se réécrit $8(x + \frac{3}{2})(x - \frac{5}{4}) = (2x + 3)(4x - 5)$.

9. On a $\Delta = (10)^2 - 4(1)(24) = 4 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(10)-\sqrt{4}}{2(1)} = -6$ et $x_2 = \frac{-(10)+\sqrt{4}}{2(1)} = -4$ donc le trinôme $x^2 + 10x + 24$ se réécrit $(x + 6)(x + 4)$.

10. On a $\Delta = (-7)^2 - 4(1)(12) = 1 > 0$. Le trinôme du second degré admet deux racines distinctes $x_1 = \frac{-(-7)-\sqrt{1}}{2(1)} = 3$ et $x_2 = \frac{-(-7)+\sqrt{1}}{2(1)} = 4$ donc le trinôme $x^2 - x + 12$ se réécrit $(x - 3)(x - 4)$.

Exercice 77

1. $x^2 + 7x + 12 = (x + \frac{7}{2})^2 - \frac{1}{4} = (x + \frac{7}{2} - \frac{1}{2})(x + \frac{7}{2} + \frac{1}{2}) = (x + 3)(x + 4)$

2. $x^2 - 7x + 10 = (x - \frac{7}{2})^2 - \frac{9}{4} = (x - \frac{7}{2} - \frac{3}{2})(x - \frac{7}{2} + \frac{3}{2}) = (x - 5)(x - 2)$

3. $x^2 + 2x - 15 = (x + 1)^2 - 16 = (x + 1 - 4)(x + 1 + 4) = (x - 3)(x + 5)$

4. $x^2 - 5x - 14 = (x - \frac{5}{2})^2 - \frac{81}{4} = (x - \frac{5}{2} - \frac{9}{2})(x - \frac{5}{2} + \frac{9}{2}) = (x - 7)(x + 2)$

5. $x^2 + 14x + 48 = (x + 7)^2 - 1 = (x + 7 - 1)(x + 7 + 1) = (x + 6)(x + 8)$

6. $x^2 - 15x + 36 = (x - \frac{15}{2})^2 - \frac{81}{4} = (x - \frac{15}{2} - \frac{9}{2})(x - \frac{15}{2} + \frac{9}{2}) = (x - 12)(x - 3)$

Exercice 78

1. $2x^3 - 8x^2 + 6x = 2x(x^2 - 4x + 3) = 2x[(x - 2)^2 - 1] = 2x(x - 2 - 1)(x - 2 + 1) = 2x(x - 3)(x - 1)$

2. $6x^4 + 9x^3 + 3x^2 = 6x^2(x^2 + \frac{3}{2}x + \frac{1}{2}) = 6x^2[(x + \frac{3}{4})^2 - \frac{1}{16}] = 6x^2(x + \frac{3}{4} - \frac{1}{4})(x + \frac{3}{4} + \frac{1}{4}) = 6x^2(x + \frac{1}{2})(x + 1)$

3. $9x^4 + 6x^2 + 1 = 9(x^4 + \frac{2}{3}x^2 + \frac{1}{9}) = 9(x^2 + \frac{1}{3})^2$

4. $x^4 - 2x^2 + 1 = (x^2 - 1)^2$

5. $x^3 + 3x^2 + 2x = x(x^2 + 3x + 2) = x[(x + \frac{3}{2})^2 - \frac{1}{4}] = x(x + \frac{3}{2} - \frac{1}{2})(x + \frac{3}{2} + \frac{1}{2}) = x(x + 1)(x + 2)$

6. $16x^3 + 4x^2y - 2xy^2 = 16x(x^2 + \frac{1}{4}xy - \frac{1}{8}y^2) = 16x[(x + \frac{1}{8}y)^2 - \frac{9}{64}y^2] = 16x(x + \frac{1}{8}y - \frac{3}{8}y)(x + \frac{1}{8}y + \frac{3}{8}y)$
   $= 16x(x - \frac{1}{4}y)(x + \frac{1}{2}y)$

7. $(2x - 5)^2 - 4(x - 1)^2 = (2x - 5)^2 - [2(x - 1)]^2 = (2x - 5 - 2(x - 1))(2x - 5 + 2(x - 1)) = -3(4x - 7)$

8. $x^8 - y^8 = (x^4 - y^4)(x^4 + y^4) = (x^2 - y^2)(x^2 + y^2)(x^4 + y^4) = (x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

Exercice 79

1. (a) Il suffit de développer $(a + b)(a^2 - ab + b^2)$.

   (b) Il suffit de développer $(a - b)(a^2 + ab + b^2)$.

2. (a) D'après la formule précédente, $x^3 + 64 = (x + 4)(x^2 - 4x + 16)$.

   (b) D'après la formule précédente, $8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$.

   (c) D'après la formule précédente, $27x^3 - 8y^3 = (3x - 2y)(9x^2 + 6xy + 4y^2)$.

Exercice 80

1. $\frac{5x^2}{20x^3} = \frac{1}{4x}$ si $x \neq 0$.

2. $\frac{12x^3y^2}{16xy^3} = \frac{3x^2}{4y}$ si $x, y \neq 0$.

3. $\frac{5x+10y}{5x-10y} = \frac{x+2y}{x-2y} = \frac{x-2y+4y}{x-2y} = 1 + \frac{4y}{x-2y}$ si $x \neq 2y$.

4. $\frac{2x^2+3x}{5x^2+10x} = \frac{x(2x+3)}{5x(x+2)} = \frac{2x+3}{5(x+2)}$ si $x \neq -2$ et $x \neq 0$.

5. $\frac{6x^3+4x^2}{9x^2+6x} = \frac{2x^2(3x+2)}{3x(3x+2)} = \frac{2x}{3}$ si $x \neq 0$ et $x \neq -\frac{2}{3}$.

6. $\frac{x^2+3x+2}{x^2+x-2} = \frac{(x+1)(x+2)}{(x-1)(x+2)} = \frac{x+1}{x-1}$ si $x \neq 1$ et $x \neq -2$.

7. $\frac{2x^2-x-6}{2x^2+5x+3} = \frac{(x-2)(2x+3)}{(x+1)(2x+3)} = \frac{x-2}{x+1}$ si $x \neq -\frac{3}{2}$ et $x \neq -1$.

8. $\frac{x^2-9}{x^2+6x+9} = \frac{(x-3)(x+3)}{(x+3)^2} = \frac{x-3}{x+3}$ si $x \neq -3$.

Exercice 81

1. $\frac{x^2-5x}{x^2-25} = \frac{x(x-5)}{(x-5)(x+5)} = \frac{x}{x+5}$ si $x \neq -5$ et $x \neq 5$.

2. $\frac{x^4-1}{x^3-x} = \frac{(x^2-1)(x^2+1)}{x(x^2-1)} = \frac{x^2+1}{x}$ si $x \neq -1$, $x \neq 0$ et $x \neq 1$.

3. $\frac{(x+2)^2-9}{x^2-25} = \frac{(x+2-3)(x+2+3)}{(x-5)(x+5)} = \frac{(x-1)(x+5)}{(x-5)(x+5)} = \frac{x-1}{x-5}$ si $x \neq -5$ et $x \neq 5$.

4. $\frac{x^2+2x-15}{x^2-9} = \frac{(x+5)(x-3)}{(x-3)(x+3)} = \frac{x+5}{x+3}$ si $x \neq -3$ et $x \neq 3$.

5. $\frac{2x^2+7x+3}{4x^2-1} = \frac{(x+3)(2x+1)}{(2x-1)(2x+1)} = \frac{x+3}{2x-1}$ si $x \neq -\frac{1}{2}$ et $x \neq \frac{1}{2}$.

6. $\frac{x^2+5x+6}{x^2+x-2} = \frac{(x+2)(x+3)}{(x+2)(x-1)} = \frac{x+3}{x-1}$ si $x \neq -2$ et $x \neq 1$.

7. $\frac{x^2-x-6}{2x^2-5x-3} = \frac{(x+2)(x-3)}{(2x+1)(x-3)} = \frac{x+2}{2x+1}$ si $x \neq -\frac{1}{2}$ et $x \neq 3$.

8. $\frac{x^2-2x+1}{x^2-1} = \frac{(x-1)^2}{(x-1)(x+1)} = \frac{x-1}{x+1}$ si $x \neq -1$ et $x \neq 1$.

Exercice 82

1. $\frac{x^2-y^2}{x^2-2xy+y^2} = \frac{(x-y)(x+y)}{(x-y)^2} = \frac{x+y}{x-y}$ si $x \neq y$.

2. $\frac{x^3-x}{x^3-2x^2+x} = \frac{x(x^2-1)}{x(x^2-2x+1)} = \frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1}$ si $x \neq 1$.

3. $\frac{x^2-1}{(x+1)(2x^2-2x)} = \frac{(x-1)(x+1)}{(x+1)2x(x-1)} = \frac{1}{2x}$ si $x \neq -1$, $x \neq 0$ et $x \neq 1$.

4. $\frac{(x+1)^2+x^2+x}{4x^2-1} = \frac{2x^2+3x+1}{(2x-1)(2x+1)} = \frac{(2x+1)(x+1)}{(2x-1)(2x+1)} = \frac{x+1}{2x-1}$ si $x \neq -\frac{1}{2}$ et $x \neq \frac{1}{2}$.

Exercice 83

1. $\frac{12x}{5} \times \frac{10}{3x^2} = \frac{8}{x}$ si $x \neq 0$.

2. $\frac{2x}{x-1} \times \frac{x-1}{5} = \frac{2x}{5}$ si $x \neq 1$.

3. $\frac{x-5}{x+1} \times \frac{x-2}{x-5} = \frac{x-2}{x+1}$ si $x \neq -1$ et $x \neq 5$.

4. $\frac{2x}{2x+1} \times \frac{2x+1}{3x^2} = \frac{2}{3x}$ si $x \neq -\frac{1}{2}$ et $x \neq 0$.

5. $\frac{3x-1}{2x-3} \times \frac{4x-6}{3x-1} = 2$ si $x \neq \frac{1}{3}$ et $x \neq \frac{3}{2}$.

6. $\frac{3x-15}{2x+10} \times \frac{4x+20}{6x-30} = 1$ si $x \neq -5$ et $x \neq 5$.

Exercice 84

1. $\frac{2x-4}{x+3} \times \frac{x^2+6x+9}{x^2-4} = \frac{2(x-2)(x+3)^2}{(x+3)(x-2)(x+2)} = \frac{2(x+3)}{x+2}$ si $x \neq -3$, $x \neq -2$ et $x \neq 2$.

2. $\frac{x^2-1}{x+3} \times \frac{x-3}{x^2-4x+3} = \frac{(x-1)(x+1)(x-3)}{(x+3)(x-1)(x-3)} = \frac{x+1}{x+3}$ si $x \neq -3$, $x \neq 1$ et $x \neq 3$.

3. $\frac{2x+3}{x-1} \times \frac{x^2+2x-3}{2x^2-x-6} = \frac{(2x+3)(x-1)(x+3)}{(x-1)(x-2)(2x+3)} = \frac{x+3}{x-2}$ si $x \neq -\frac{3}{2}$, $x \neq 1$ et $x \neq 2$.

4. $\frac{2x^2+6x}{x+4} \times \frac{x^2+8x+16}{5x^2+15x} = \frac{2x(x+3)(x+4)^2}{(x+4)5x(x+3)} = \frac{2(x+4)}{5}$ si $x \neq -4$, $x \neq -3$ et $x \neq 0$.

5. $\frac{x^2+x-6}{x^2-4x-5} \times \frac{x^2+3x+2}{x^2-6x+8} = \frac{(x-2)(x+3)(x+1)(x+2)}{(x+1)(x-5)(x-2)(x-4)} = \frac{(x+3)(x+2)}{(x-5)(x-4)}$ si $x \neq -1$, $x \neq 2$, $x \neq 4$ et $x \neq 5$.

6. $\frac{2x^2-3x-2}{x^2-1} \times \frac{x-1}{2x+1} = \frac{(x-2)(2x+1)(x-1)}{(x-1)(x+1)(2x+1)} = \frac{x-2}{x+1}$ si $x \neq -1$, $x \neq -\frac{1}{2}$ et $x \neq 1$.

Exercice 85

1. $\frac{x^2-1}{x+2} \div \frac{x-1}{3x+6} = \frac{x^2-1}{x+2} \times \frac{3x+6}{x-1} = \frac{(x-1)(x+1)3(x+2)}{(x+2)(x-1)} = 3(x+1)$ si $x \neq -2$ et $x \neq 1$.

2. $\frac{x^2-x-2}{x^2-x-6} \div \frac{x+1}{x+2} = \frac{x^2-x-2}{x^2-x-6} \times \frac{x+2}{x+1} = \frac{(x+1)(x-2)(x+2)}{(x+2)(x-3)(x+1)} = \frac{x-2}{x-3}$ si $x \neq -2$, $x \neq -1$ et $x \neq 3$.

3. $\frac{3x^2+8x-3}{x^2+x-6} \div \frac{2x+1}{x-2} = \frac{3x^2+8x-3}{x^2+x-6} \times \frac{x-2}{2x+1} = \frac{(x+3)(3x-1)(x-2)}{(x-2)(x+3)(2x+1)} = \frac{3x-1}{2x+1}$ si $x \neq -3$, $x \neq -\frac{1}{2}$ et $x \neq 2$.

4. $\frac{2x^2+2x}{x+5} \div \frac{2x^3-2x}{x^2+10x+25} = \frac{2x^2+2x}{x+5} \times \frac{x^2+10x+25}{2x^3-2x} = \frac{2x(x+1)(x+5)^2}{2x(x-1)(x+1)} = \frac{(x+5)^2}{x-1}$ si $x \neq -5$, $x \neq -1$, $x \neq 0$ et $x \neq 1$.

5. $\frac{2x-4}{x^2+6x+9} \div \frac{x^2-4}{x^2-9} = \frac{2x-4}{x^2+6x+9} \times \frac{x^2-9}{x^2-4} = \frac{2(x-2)(x-3)(x+3)}{(x+3)^2(x-2)(x+2)} = \frac{2(x-3)}{(x+2)(x+3)}$ si $x \neq -3$, $x \neq -2$, $x \neq 2$ et $x \neq 3$.

6. $\frac{x^4-1}{x^2+1} \div \frac{x^2-1}{x+2} = \frac{x^4-1}{x^2+1} \times \frac{x+2}{x^2-1} = \frac{(x-1)(x+1)(x^2+1)(x+2)}{(x^2+1)(x-1)(x+1)} = x+2$ si $x \neq -2$, $x \neq -1$ et $x \neq 1$.

1. $\frac{2}{x} + \frac{5}{y} = \frac{5x+2y}{xy}$

2. $\frac{3}{x} - \frac{2}{x^2} = \frac{3x-2}{x^2}$

3. $\frac{4x}{y^2} - \frac{4x-y}{xy} = \frac{4x^2-4xy+y^2}{xy^2}$

4. $\frac{2x+3y}{9x} + \frac{2x-3y}{6y} = \frac{4xy+6y^2+6x^2-9xy}{18xy} = \frac{6x^2-5xy+6y^2}{18xy}$

Exercice 87

1. $\frac{x}{x^2-9} - \frac{1}{2x-6} = \frac{x}{(x-3)(x+3)} - \frac{1}{2(x-3)} = \frac{2x-(x+3)}{2(x-3)(x+3)} = \frac{x-3}{2(x-3)(x+3)} = \frac{1}{2(x+3)}$

2. $\frac{1}{x+5} + \frac{1}{x-5} = \frac{(x-5)+(x+5)}{(x+5)(x-5)} = \frac{2x}{(x+5)(x-5)}$

3. $\frac{1}{x^2-2x+1} + \frac{1}{x-1} = \frac{1}{(x-1)^2} + \frac{1}{x-1} = \frac{1+(x-1)}{(x-1)^2} = \frac{x}{(x-1)^2}$

4. $\frac{2a}{3a-15} + \frac{4a}{2a-10} = \frac{2a}{3(a-5)} + \frac{4a}{2(a-5)} = \frac{4a+12a}{6(a-5)} = \frac{16a}{6(a-5)} = \frac{8a}{3(a-5)}$

Exercice 88

1. $\frac{2x+1}{2} - \frac{3x-1}{5} = \frac{5(2x+1)-2(3x-1)}{10} = \frac{4x+7}{10}$

2. $\frac{5}{x-2} + \frac{3}{x+3} = \frac{5(x+3)+3(x-2)}{(x-2)(x+3)} = \frac{8x+9}{(x-2)(x+3)}$

3. $\frac{x+1}{x-2} - \frac{x+2}{x-1} = \frac{(x+1)(x-1)-(x-2)(x+2)}{(x-2)(x-1)} = \frac{(x^2-1)-(x^2-4)}{(x-2)(x-1)} = \frac{3}{(x-2)(x-1)}$

4. $\frac{x+3}{x-3} - \frac{x-3}{x+3} = \frac{(x+3)^2-(x-3)^2}{(x-3)(x+3)} = \frac{12x}{(x-3)(x+3)}$

5. $\frac{x+1}{x^2-4} - \frac{x-2}{x+2} = \frac{x+1}{(x-2)(x+2)} - \frac{x-2}{x+2} = \frac{(x+1)-(x-2)^2}{(x-2)(x+2)} = \frac{-x^2+5x-3}{(x-2)(x+2)}$

6. $\frac{x+1}{x^2-2x+1} + \frac{1}{x^2-1} = \frac{x+1}{(x-1)^2} + \frac{1}{(x-1)(x+1)} = \frac{(x+1)^2+(x-1)}{(x-1)^2(x+1)} = \frac{x^2+3x}{(x-1)^2(x+1)} = \frac{x(x+3)}{(x-1)^2(x+1)}$

Exercice 89

1. $\frac{1}{x+2} - \frac{1}{x-2} + \frac{x+6}{x^2-4} = \frac{1}{x+2} - \frac{1}{x-2} + \frac{x+6}{(x-2)(x+2)} = \frac{(x-2)-(x+2)+(x+6)}{(x-2)(x+2)} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$

2. $\frac{5}{x^2-2x} + \frac{4}{x^2-4} - \frac{2}{x^2+2x} = \frac{5}{x(x-2)} + \frac{4}{(x-2)(x+2)} - \frac{2}{x(x+2)} = \frac{5(x+2)+4x-2(x-2)}{x(x-2)(x+2)} = \frac{7x+14}{x(x+2)(x-2)} = \frac{7}{x(x-2)}$

Exercice 90

1. Comme $\frac{1}{x^2+2x-3} = \frac{1}{(x-1)(x+3)}$, on cherche $A, B \in \mathbb{R}$ tels que

$$\frac{1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)} = \frac{(A+B)x+(3A-B)}{(x-1)(x+3)}.$$

Par identification, on a $\begin{cases} A+B = 0 \\ 3A-B = 1 \end{cases} \Leftrightarrow \begin{cases} A = \frac{1}{4} \\ B = -\frac{1}{4} \end{cases}$ donc $\frac{1}{x^2+2x-3} = \frac{1}{4(x-1)} - \frac{1}{4(x+3)}$.

2. Comme $\frac{5x^2}{x^2-3x-4} = \frac{5(x^2-3x-4)+5(3x+4)}{x^2-3x-4} = 5 + \frac{5(3x+4)}{x^2-3x-4} = 5 + \frac{15x+20}{(x+1)(x-4)}$, on cherche $A, B \in \mathbb{R}$ tels que

$$\frac{15x+20}{x^2-3x-4} = \frac{A}{x+1} + \frac{B}{x-4} = \frac{A(x-4)+B(x+1)}{(x+1)(x-4)} = \frac{(A+B)x+(B-4A)}{(x+1)(x-4)}.$$

Par identification, on a $\begin{cases} A+B = 15 \\ B-4A = 20 \end{cases} \Leftrightarrow \begin{cases} A = -1 \\ B = 16 \end{cases}$ donc $\frac{5x^2}{x^2-3x-4} = 5 - \frac{1}{x+1} + \frac{16}{x-4}$.

3. On trouve aisément que $\frac{x^2+1}{x+1} = \frac{x(x+1)-x+1}{x+1} = x - \frac{x-1}{x+1} = x - \frac{(x+1)-2}{x+1} = x - 1 + \frac{2}{x+1}$.