Categorical properties of the complex numbers

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Category of Hilbert spaces

Finite-dimensional quantum mechanics takes place in **Hilb**, with

- finite-dimensional complex Hilbert spaces as objects
- continuous linear maps as morphisms

Symmetric monoidal structure, tensor unit is $\mathbb{C}$
Can access $\mathbb{C}$ as the *scalars*, $\mathrm{Hom}(I, I)$

The field $\mathbb{C}$ is vitally important for quantum theory
What are its categorical properties?

Aim. Find a set of properties for a monoidal category which imply that the scalars are 'similar' to $\mathbb{C}$.

Strategy. Steal the properties of **Hilb**!

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The †-functor

Definition. A †-*category* is a category $\mathbf{C}$ equipped with a †-*functor*, a functor $\dagger : \mathbf{C} \longrightarrow \mathbf{C}$ which is

- contravariant
- involutive
- identity on objects

Example: taking the adjoint of a map between Hilbert spaces

$$f : A \longrightarrow B \;\longmapsto\; f^\dagger : B \longrightarrow A$$

Gives a †-functor $\dagger : \mathbf{Hilb} \longrightarrow \mathbf{Hilb}$.

A morphism $f : A \longrightarrow B$ is
an *isometry* if it satisfies $f^\dagger \circ f = \mathrm{id}_A$
unitary if it satisfies $f^\dagger \circ f = \mathrm{id}_A$ and $f \circ f^\dagger = \mathrm{id}_B$
A morphism $f : A \longrightarrow A$ is *self-adjoint* if $f^\dagger = f$

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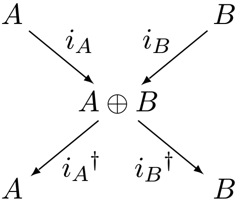
Definition. A †-*biproduct* is a biproduct for which the injections and projections are related by the †-functor.



$$i_A; i_A{}^\dagger = \mathrm{id}_A \quad i_B; i_A{}^\dagger = 0_{B,A}$$
$$i_A; i_B{}^\dagger = 0_{A,B} \quad i_B; i_B{}^\dagger = \mathrm{id}_B$$

$$i_A{}^\dagger; i_A + i_B{}^\dagger; i_B = \mathrm{id}_{A\oplus B}$$

Unique up to unique unitary isomorphism.

†-Equalisers

Equalisers are always monic.
Idea: require these to be isometries. This is possible in **Hilb**.

Definition (Selinger). In a †-category, a †-*equaliser* is an equaliser $e : E \longrightarrow A$ such that $e; e^\dagger = \mathrm{id}_E$.
Unique up to unique unitary isomorphism.

In a category with a zero object, a †-functor is *nondegenerate* if

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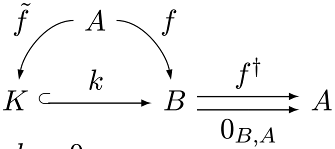
Lemma. In a †-category with a zero object and finite †-equalisers, the †-functor is nondegenerate.



$$f = \tilde{f}; k = \tilde{f}; k; k^\dagger; k = f; k^\dagger; k = 0_{A,K}; k = 0_{A,B}.$$

Cancellable addition

What do we get if we combine †-biproducts and †-equalisers?

Lemma. In a †-category with finite †-biproducts and †-equalisers, hom-set addition is *cancellable*: for all f, g, h in the same hom-set,

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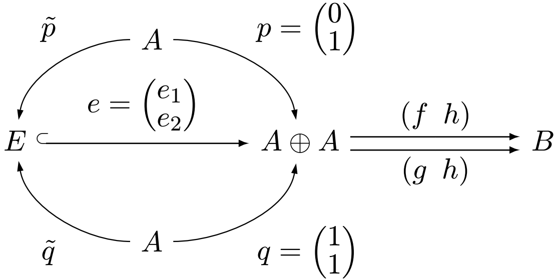

Examples: **Hilb** has †-biproducts and †-equalisers
$\Rightarrow$ has cancellable addition
Rel has †-biproducts, lacks cancellable addition
$\Rightarrow$ lacks †-equalisers

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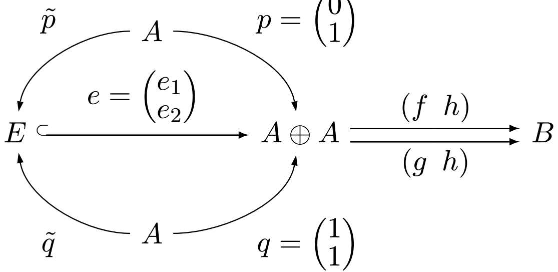

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What about the complex numbers?

Definition. A *monoidal †-category* is a monoidal category which is also a †-category, such that all of the structural isomorphisms are unitary.

In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, the scalars will be

- commutative
- additively cancellable

We're on the right track!

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Guess 1. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, the scalars are 'like' the complex numbers.

Problem. Could take the cartesian product of two such categories to get another such category — for example, **Hilb** × **Hilb** has scalars given by *pairs* of complex numbers (a, b), with algebra

$$(a, b) + (c, d) = (a + c, b + d) \qquad (a, b) \cdot (c, d) = (ac, bd)$$

Guess 2. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, *for which the monoidal unit has no proper subobjects*, the scalars are 'like' the complex numbers.

How can we make this precise?

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Main result

Theorem. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, for which the monoidal unit has no proper subobjects, *the semiring of scalars embeds into a field of characteristic 0.*

To prove this, use the fact that the subsemirings of fields are exactly those semirings which

- are commutative;
- have cancellable addition ($a + c = b + c \Rightarrow a = b$);
- are such that if $a \neq b$, then
$$Aa + Bb = Ab + Ba \Rightarrow A = B.$$

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'No subobjects of I' criterion required for these properties

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Theorem. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, for which the monoidal unit has no proper subobjects, the semiring of scalars embeds into a field of characteristic 0.

Fact. The subfields of the complex numbers are exactly the fields of characteristic 0 with at most continuum cardinality.

So...

Corollary. In a nontrivial monoidal †-category with †-biproducts and †-equalisers, for which the monoidal unit has no proper subobjects, *and such that the scalars have at most continuum cardinality*, the scalars are a subsemiring of the complex numbers.

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What about the real numbers?

The scalars have an involution given by the †-functor:

$$a : I \longrightarrow I \;\longmapsto\; a^\dagger : I \longrightarrow I$$

In **Hilb**, the real numbers are the self-adjoint scalars:

$$\mathbb{R} = \{a : \mathbb{C} \longrightarrow \mathbb{C} \,|\, a^\dagger = a\}$$

They have lots of great properties, including a *total order.*

We can generalise this.

Theorem. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, for which the monoidal unit has no proper subobjects, *the semiring of self-adjoint scalars admits a total order.*

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Fact. The subfields of the real numbers are exactly the fields which admit an *Archimedean ordering*, one for which every element is between two rational numbers.

Corollary. Every field that admits an Archimedean ordering has at most the cardinality of the continuum.

This lets us rephrase our complex numbers theorem:
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Open questions

Archimedeanity

- Has an algebraic definition, should be tractable categorically
- Gives an algebraic way to control the cardinality
- Could lead to a *continuous* embedding of the scalars into $\mathbb{C}$

General †-limits

- †-biproducts and †-equalisers seem to be useful
- Given the existence theorem for limits, maybe there is a general theory of †-limits to be uncovered?

Embedding the entire category into Hilb

- Need a categorical way to ensure that every object $\simeq I^{\oplus N}$

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- Given the existence theorem for limits, maybe there is a general theory of †-limits to be uncovered?

Embedding the entire category into Hilb

- Need a categorical way to ensure that every object $\simeq I^{\oplus N}$

Open questions

Archimedeanity

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Thank you!

Theorem. In a nontrivial monoidal †-category with finite †-biproducts and †-equalisers, for which the monoidal unit has no proper subobjects, the semiring of scalars embeds into an involutive field of characteristic 0 with orderable fixed field.