Making weak maps compose strictly

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Outline

Motivation

Weak maps of bicategories

Weak maps of tricategories

Weak maps of weak ω-categories

(NB: Talk notes available at http://www.dpmms.cam.ac.uk/∼rhgg2)

Motivation

Consider a category $\mathscr{C}$ with:

- *Objects* being higher-dimensional ——s;
- *Morphisms* being strict structure-preserving maps.

Would like to derive $\mathscr{C}_{\mathrm{wk}}$ with:

- Same objects;
- Morphisms being *weak* structure-preserving maps.

Motivation

Idea from homotopy theory: identify

$$\textit{weak maps } X \to Y \qquad \text{with} \qquad \textit{strict maps } X' \to \tilde{Y}$$

where:

- X' is a *cofibrant replacement* for X;
- $\tilde{Y}$ is a *fibrant replacement* for Y.

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- A strict map $X \to \tilde{Y}$ is a map *which preserves the differential only up to homotopy*;
- A strict map $X' \to \tilde{Y}$ (= weak map $X \to Y$) is a map *which preserves the R-module structure and the differential only up to homotopy*.

Example: $f/\mathbf{Cat}/Z$

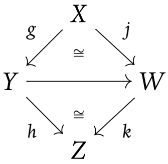
Let $f\colon X \to Z$ in **Cat**. Can form the *interval category* $f/\mathbf{Cat}/Z$:

- *Objects* are $X \xrightarrow{g} Y \xrightarrow{h} Z$ with $hg = f$;
- *Morphisms* are commutative diamonds:

$$\begin{array}{ccccc} & & X & & \\ & {\scriptstyle g}\swarrow & & \searrow{\scriptstyle j} & \\ Y & & \longrightarrow & & W. \\ & {\scriptstyle h}\searrow & & \swarrow{\scriptstyle k} & \\ & & Z & & \end{array}$$

Example: $f/\mathbf{Cat}/Z$

Corresponding *weak maps* should be pseudo-commutative diamonds:



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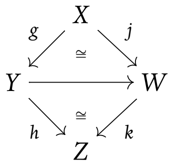
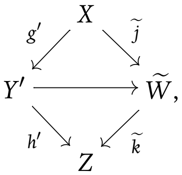


... and these are precisely strict maps



where:

Example: $f/\mathbf{Cat}/Z$

… fibrant replacement of $X \xrightarrow{j} W \xrightarrow{k} Z$ is:

$$X \xrightarrow{\lambda_k \circ j} W \downarrow_{\cong} k \xrightarrow{\rho_k} Z$$

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… fibrant replacement of $X \xrightarrow{j} W \xrightarrow{k} Z$ is:

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and cofibrant replacement of $X \xrightarrow{g} Y \xrightarrow{h} Z$ is:

$$X \xrightarrow{l_g} g \uparrow_{\cong} Y \xrightarrow{h \circ r_g} Z.$$

How to compose weak maps?

Idea from category theory:

- Cofibrant replacement should be a *comonad* $(-)' \colon \mathscr{C} \to \mathscr{C}$;
- Fibrant replacement should be a *monad* $\widetilde{(-)} \colon \mathscr{C} \to \mathscr{C}$;
- There should be a *distributive law* $d_X \colon (\tilde{X})' \to \widetilde{X'}$.

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Now composition of weak maps is *two-sided Kleisli composition*:

$$(X' \xrightarrow{f} \tilde{Y}) \circ (Y' \xrightarrow{g} \tilde{Z}) \quad := $$
$$X' \xrightarrow{\Delta_X} X'' \xrightarrow{f'} (\tilde{Y})' \xrightarrow{d_Y} \widetilde{Y'} \xrightarrow{\tilde{g}} \tilde{\tilde{Z}} \xrightarrow{\mu_Z} \tilde{Z}.$$

Example: $f/\mathbf{Cat}/Z$

- Cofibrant replacement *is* a comonad [Grandis–Tholen 2006];
- Fibrant replacement *is* a monad [loc. cit.];
- There *is* a distributive law between them;

and corresponding Kleisli composition is what you think it is: pasting of pseudo-commutative diamonds.

In general

If a (locally presentable) category $\mathscr{C}$ has a cofibrantly generated model structure on it, then:

- Cofibrant replacement can be made a comonad [G. 2008];
- Fibrant replacement can be made a monad [loc. cit.];
- But not clear how to get a distributive law between them!

So in this talk, we focus on the case where *every object is fibrant*.

(As then we only need cofibrant replacement comonad).

Weak maps of bicategories

Consider the category $\mathbf{Bicat}_s$:

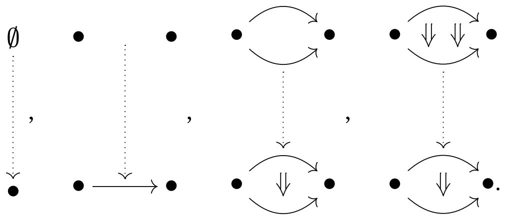
- *Objects* are bicategories;
- *Morphisms* are strict homomorphisms.

There is a cofibrantly generated model structure on $\mathbf{Bicat}_s$ [Lack, 2004], wherein:

- Weak equivalences are biequivalences;
- Every object is fibrant.

What are the corresponding weak maps?

First we describe cofibrant replacement comonad $(-)'$. It is generated by the following set of maps in $\mathbf{Bicat}_s$:



Explicitly, if $\mathscr{B}$ is a bicategory, then $\mathscr{B}'$ is given as follows:

- Ignore the 2-cells and form the free bicategory $FU\mathscr{B}$ on the underlying 1-graph of $\mathscr{B}$;

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- Factorise the counit map $\varepsilon\colon FU\mathscr{B} \to \mathscr{B}$ as

$$FU\mathscr{B} \xrightarrow{a} \mathscr{B}' \xrightarrow{b} \mathscr{B}$$

  where a is bijective on objects and 1-cells and b is locally fully faithful.

(NB: this is the *flexible replacement* of [Blackwell-Kelly-Power 1989]).

Proposition (Coherence for homomorphisms)

The co-Kleisli category of $(-)' \colon \mathbf{Bicat}_s \to \mathbf{Bicat}_s$ *is isomorphic to the category* $\mathbf{Bicat}$ *of bicategories and homomorphisms.*

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Proof.

- First define a comonad H on $\mathbf{Bicat}_s$ such that $\mathbf{Kl}(H) \cong \mathbf{Bicat}$ by construction;
- Then show that $H \cong (-)'$ as comonads.

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Explicitly, given bicategory $\mathscr{B}$, we form $H\mathscr{B}$ as follows:

Start with $FU\mathscr{B}$ as above. Given $f \colon X \to Y$ in $\mathscr{B}$, write $[f] \colon X \to Y$ for corresponding generator in $FU\mathscr{B}$.

Now adjoin 2-cells to $FU\mathcal{B}$ as follows:

- For each $\alpha\colon f \Rightarrow g$ in $\mathcal{B}$, a 2-cell
$$[\alpha]\colon [f] \Rightarrow [g];$$
- For each $X \in \mathcal{B}$, a 2-cell
$$\eta_X\colon \mathrm{id}_X \Rightarrow [\mathrm{id}_X];$$
- For each $X \xrightarrow{f} Y \xrightarrow{g} Z \in \mathcal{B}$, a 2-cell
$$\mu_{g,f}\colon [g] \circ [f] \Rightarrow [g \circ f];$$

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And quotient out the 2-cells by equations making:

- $[-]$ be functorial on 2-cells;
- $\mu_{g,f}$ be natural in g and f;
- The $\mu_{g,f}$'s and η_X's satisfy the unit and associativity laws.

The result of this is $H\mathcal{B}$.

- By construction, maps $H\mathscr{B} \to \mathscr{C}$ are in bijection with homomorphisms $\mathscr{B} \to \mathscr{C}$.

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- We can now make H into a comonad so that $\mathbf{Kl}(H) \cong \mathbf{Bicat}$ (comonad structure on H is combinatorial essence of composition of homomorphisms—compare [Hess-Parent-Scott 2006]).
- Finally, we show that $H \cong (-)'$ as comonads (a *normalization proof*).

Weak maps of tricategories

Consider the category $\mathbf{Tricat}_s$:

- *Objects* are tricategories;
- *Morphisms* are strict homomorphisms.

We use an algebraic definition of tricategory, so $\mathbf{Tricat}_s$ is l.f.p. and in particular cocomplete.

Weak maps of tricategories

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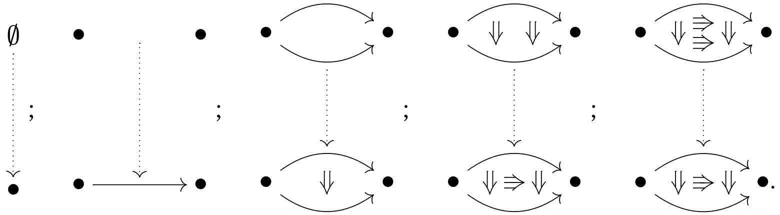
We use an algebraic definition of tricategory, so $\mathbf{Tricat}_s$ is l.f.p. and in particular cocomplete.

No-one has written down the cofibrantly generated model structure on $\mathbf{Tricat}_s$ yet, but it should have:

- Weak equivalences being triequivalences;
- Every object being fibrant.

Can we describe the corresponding weak maps?

Yes: because we can describe the cofibrant replacement comonad $(-)'$. It's generated by the following set of maps in $\mathbf{Tricat}_s$:



Explicitly, if $\mathcal{T}$ is a tricategory, then $\mathcal{T}'$ is given as follows:

- Ignore the 2- and 3-cells and form the free tricategory $FU\mathcal{T}$ on the underlying 1-graph of $\mathcal{T}$. Write $\varepsilon\colon FU\mathcal{T} \to \mathcal{T}$ for the counit map.

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- For each pair of 1-cells $f, g\colon X \to Y$ in $FU\mathcal{T}$ and each 2-cell $\alpha\colon \varepsilon(f) \Rightarrow \varepsilon(g)$ in $\mathcal{T}$, adjoin a 2-cell $(f, g, \alpha)\colon f \Rightarrow g$ to $FU\mathcal{T}$. Call the result $\mathcal{T}^{\#}$, and write $\varepsilon^{\#}\colon \mathcal{T}^{\#} \to \mathcal{T}$ for the induced counit.

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- Factorise $\varepsilon^\#$ as

$$\mathcal{T}^\# \xrightarrow{a} \mathcal{T}' \xrightarrow{b} \mathcal{T}$$

  where a is bijective on 0-, 1- and 2-cells and b is locally locally fully faithful.

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- *A priori* quite surprising, because trihomomorphisms à la [Gordon-Power-Street 1995] do not compose associatively: there *is* no category of tricategories and (ordinary) trihomomorphisms.
- So what *do* these new weak morphisms look like? Can they really be as weak as trihomomorphisms?

Definition

An *unbiased trihomomorphism* $F\colon \mathcal{T} \to \mathcal{U}$ is given by:

- For each object $X \in \mathcal{T}$, an object $FX \in \mathcal{U}$;
- For each 1-cell $f\colon X \to Y \in \mathcal{T}$, a 1-cell $Ff\colon FX \to FY$ in $\mathcal{U}$;

Plus...

- For every bracketed pasting diagram

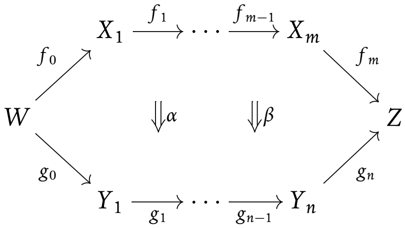
$$A \;:=\; \begin{tikzcd} & X_1 \arrow[r, "f_1"] & \cdots \arrow[r, "f_{m-1}"] & X_m \arrow[dr, "f_m"] & \\ W \arrow[ur, "f_0"] \arrow[dr, "g_0"'] & & \Downarrow \alpha & & Z \\ & Y_1 \arrow[r, "g_1"'] & \cdots \arrow[r, "g_{n-1}"'] & Y_n \arrow[ur, "g_n"'] & \end{tikzcd}$$

in $\mathcal{T}$, a 2-cell

$$\begin{tikzcd} & FX_1 \arrow[r, "Ff_1"] & \cdots \arrow[r, "Ff_{m-1}"] & FX_m \arrow[dr, "Ff_m"] & \\ FW \arrow[ur, "Ff_0"] \arrow[dr, "Fg_0"'] & & \Downarrow F(A) & & FZ \\ & FY_1 \arrow[r, "Fg_1"'] & \cdots \arrow[r, "Fg_{n-1}"'] & FY_n \arrow[ur, "Fg_n"'] & \end{tikzcd}$$

in $\mathcal{U}$.

- For every pair of bracketed pasting diagrams A, B with the same boundary in $\mathcal{T}$; i.e.,

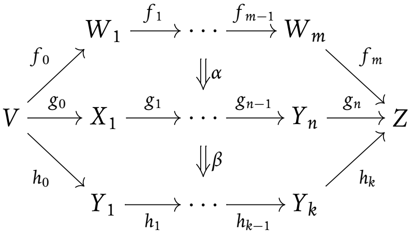


  and for every 3-cell $\Gamma\colon \alpha \Rrightarrow \beta$ between them, a 3-cell

$$F\Gamma\colon F(A) \Rrightarrow F(B)$$

  in $\mathcal{U}$.

- For every pair of composable bracketed pasting diagrams A, B in $\mathcal{T}$; i.e.,

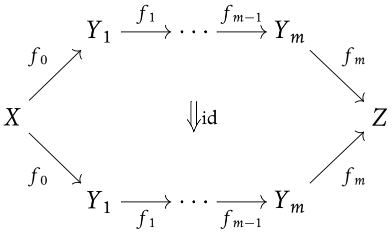


a 3-cell

$$\mu_{A,B}\colon F(B)\cdot F(A) \Rrightarrow F(B\cdot A)$$

in $\mathcal{U}$.

- For every identity pasting diagram $\mathrm{id}_{\{f_i\}}$, i.e.

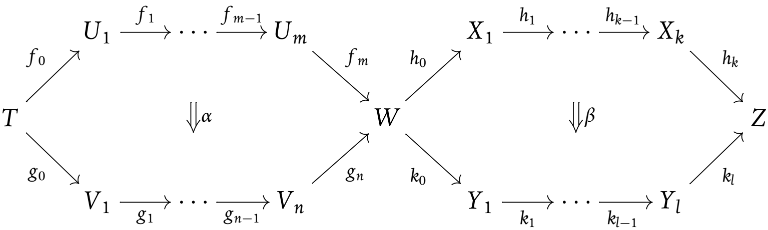


in $\mathcal{T}$, a 3-cell

$$\eta_{\{f_i\}} \colon \mathrm{id}_{\{Ff_i\}} \Rrightarrow F(\mathrm{id}_{\{f_i\}})$$

in $\mathcal{U}$.

- For every pair A, B of horizontally composable bracketed pasting diagrams:



in $\mathcal{T}$, a 3-cell

$$\gamma_{A,B} \colon F(B) \otimes F(A) \Rrightarrow F(B \otimes A)$$

in $\mathcal{U}$.

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Proposition (Coherence for unbiased trihomomorphisms)

The co-Kleisli category of $(-)' \colon \mathbf{Tricat}_s \to \mathbf{Tricat}_s$ *is isomorphic to the category* **UTricat** *of tricategories and unbiased trihomomorphisms.*

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Proof.

- First define a comonad H on $\mathbf{Tricat}_s$ such that $\mathbf{Kl}(H) \cong \mathbf{UTricat}$ by construction;
- Then show that $H \cong (-)'$ as comonads.

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Proposition

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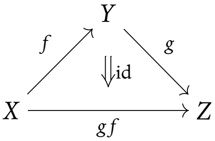
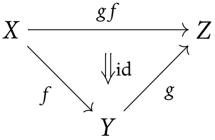
Every unbiased trihomomorphism $\mathcal{T} \to \mathcal{U}$ *gives rise to an ordinary trihomomorphism; and vice versa.*

E.g., given an unbiased trihomomorphism $F\colon \mathcal{T} \to \mathcal{U}$ let us show that it preserves 1-cell composition up to equivalence.

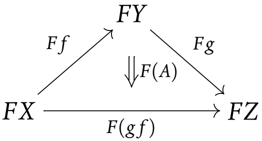
Let $X \xrightarrow{f} Y \xrightarrow{g} Z$ in $\mathcal{T}$. We have bracketed pasting diagrams A, $A^* :=$



and



in $\mathcal{T}$, and so obtain 2-cells



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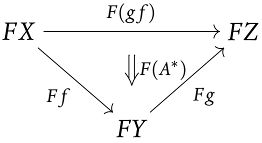

in $\mathcal{U}$. Moreover, $A \circ A^* = \mathrm{id}$ implies $F(A) \circ F(A)^* \cong \mathrm{id}$ and dually.

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Definition (G.-Gurski 2008)

Given ordinary trihomomorphisms $F, G\colon \mathcal{T} \to \mathcal{U}$, a *tricategorical icon* $\Gamma\colon F \Rightarrow G$:

- Exists only if F and G agree on 0- and 1-cells;
- Is then given by 3-cells $\Gamma_\alpha\colon F\alpha \Rrightarrow G\alpha$ for each 2-cell $\alpha \in \mathcal{T}$;
- Plus some coherence data.

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Similar definition of *unbiased tricategorical icon*.

Proposition

There is a bicategory $\mathbf{Tricat}_2$ *with:*

- *Objects being tricategories;*
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There is also a 2-category $\mathbf{UTricat}_2$ *with:*

- *Objects being tricategories;*
- *Morphisms being unbiased trihomomorphisms;*
- *2-cells being unbiased tricategorical icons.*

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The bicategory $\mathbf{Tricat}_2$ *is equivalent to the 2-category* $\mathbf{UTricat}_2$.

Proof.

- First extend the category $\mathbf{Tricat}_s$ to a 2-category, with icons as 2-cells;
- Then define a 2-comonad H on $\mathbf{Tricat}_s$ such that $\mathbf{Kl}(H) \cong \mathbf{UTricat}_2$ by construction;
- Then define a pseudo-comonad K on $\mathbf{Tricat}_s$ such that $\mathbf{Kl}(K) \cong \mathbf{Tricat}_2$ by construction;
- Finally show that $H \simeq K$ as pseudo-comonads.

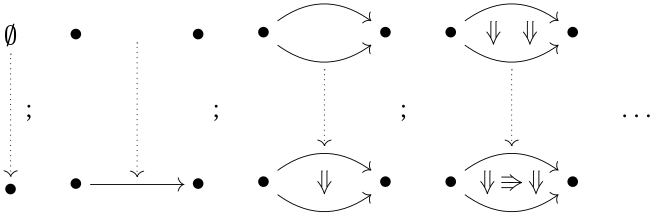
Weak maps of weak ω-categories

Consider the category ω-$\mathbf{Cat}_s$:

- *Objects* are (algebraic) weak ω-categories;
- *Morphisms* are strict homomorphisms.

We can play the same game as before to obtain a category ω-$\mathbf{Cat}$ of weak ω-categories and weak homomorphisms.

This time the cofibrant replacement comonad $(-)'$ is generated by the following set of maps in ω-$\mathbf{Cat}_s$:



Explicitly, $(-)' \colon \omega\text{-}\mathbf{Cat}_s \to \omega\text{-}\mathbf{Cat}_s$ is the comonad arising from the adjunction

$$\omega\text{-}\mathbf{Cptd} \underset{F}{\overset{U}{\underset{\longleftarrow}{\overset{\longrightarrow}{\top}}}} \omega\text{-}\mathbf{Cat}_s$$

where $\omega\text{-}\mathbf{Ctpd}_s$ is the category of ω-computads.

And by now natural to define…

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Definition

The category $\omega\text{-}\mathbf{Cat}$ of weak ω-categories and weak morphisms is the co-Kleisli category of $(-)' \colon \omega\text{-}\mathbf{Cat}_s \to \omega\text{-}\mathbf{Cat}_s$.

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