An aggregation-based algebraic multigrid method

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Abstract

Algebraic multigrid (AMG) methods are among the most efficient preconditioning techniques for large linear systems arising from discretized PDEs. In this talk, we revisit the simplest and cheapest AMG scheme, based on so-called coarsening by aggregation. We show that the basic two–grid scheme has convergence rate independent of the grid size. This is proved by developing the Fourier analysis of several aggregation–based two–grid schemes for a model anisotropic problem [1]. With a proper choice of the coarsening, the convergence rate is also uniformly bounded with respect to the anisotropy ratio. To further obtain grid independent convergence with a truly *multi*grid method, we consider so-called K–cycle multigrid [2], in which Krylov subspace acceleration is used at every level. This makes multigrid methods significantly more robust, and optimal convergence properties can be proved under mild conditions on the two–grid convergence rate. This approach is combined with an automatic aggregation scheme for which the size of most aggregates is equal to 4, independently of the problem peculiarities [3]. Numerical experiments indicate that the resulting method is efficient and can be more robust than classical AMG methods. These experiments include challenging convection–diffusion problems with high Reynolds number and varying convective flow, as well as some problems from industrial chemistry. We also discuss the parallelization of the method. The aggregation algorithm parallelizes in a natural way, and the method achieves good algorithmic scalability. Timing results indicate that satisfactory speed up can be obtained even on a relatively slow network, providing that the load per processor is kept significant.

REFERENCES

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