Stochastic Finite Elements and Fast Iterative Solvers

David J. Silvester

School of Mathematics,
University of Manchester, UK.
http://www.maths.manchester.ac.uk/djs/

Abstract

Simulation of the motion of an incompressible fluid and the transport of chemicals in porous media is a very challenging problem. In deterministic groundwater flow modelling, inputs such as material properties, boundary conditions and source terms are assumed to be known exactly. The so-called Stochastic Finite Element Method provides a framework for incorporating statistical information about spatial variability in material parameters so that more comprehensive information about the flow can be obtained. Instead of perform- ing multiple deterministic simulations, one large calculation is performed incorporating assumed statistics of the random inputs. The output can be post-processed to determine probabilistic information such as the expected concentration of a chemical in the ground- water at a nuclear waste storage site. This methodology will be reviewed in the talk with a particular focus on computational efficiency. If stochastic finite element methods are to be competitive with traditional Monte Carlo methods based on multiple realisations then we need fast and robust linear algebra techniques to solve the large indefinite systems that arise. We describe a generic block preconditioning technique for such systems with the property that the eigenvalues of the preconditioned matrices are contained in intervals that are bounded independently of the mesh size. An attractive feature is that the basis of the preconditioning is a readily available building block; namely, a scalar diffusion solve based on an algebraic multigrid V-cycle.

This is ongoing work with Oliver Ernst and Elisabeth Ullmann from Freiberg and with Catherine Powell from the University of Manchester. It is supported by the DAAD and British Council under ARC collaborative project grant 1279.