Numerical Methods for Ill-Posed Problems

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Résumé

Many problems in science and engineering require the determination of the cause of an observed effect. These problems often can be formulated as Fredholm integral equations of the first kind with a smooth kernel h,

$$\int_{\Omega} h(s,t)f(t)dt = g(s), \qquad s \in \Omega,$$

where Ω is a connected set in $\mathbf{R}^k$. The solution of problems of this kind is ill-posed in the sense of Hadamard, because the problem might not have a solution, the solution might not be unique, and the solution might not depend continuously on the right-hand side, which in applications often represents the available data. Discertization gives rise to a linear system of algebraic equations,

$$Ax = b,$$

or a least-squares problem, with a matrix of ill-determined rank. The singular values of the matrix "cluster" at the origin. In particular, the matrix is severely ill-conditioned and may be singular. The vector b is contaminated by error stemming from measurement inaccuracies, data transmission, or discretization. Due to the ill-conditioning of A and the error in b, straightforward solution of the linear system of algebraic equations does not produce a meaningful solution. Special solution techniques have to be applied. We survey numerical methods suited for small and large-scale problems. Also, nonlinear problems will be discussed.