Postdoctoral position

Counting self-avoiding polygons on lattices using sieves

Université du Littoral Côte d'Opale

Keywords: heaps of pieces, self-avoiding polygons, Lawler's loop-erased walks

Project Summary

This postdoctoral position is about the study of Self-Avoiding Polygons (SAP) on infinite regular, vertex transitive graphs known as lattices. The SAPs are simple cycles, that is cycles all of whose internal vertices are distinct. The outstanding problem in the field is to calculate the connective and universal constants which dictate the asymptotic growth of the number of SAP on lattices of length ℓ as $\ell \to \infty$. So far this problem, open since 70 years, has only been approached with the tools of probability theory and conformal field theory. Here we study this question from a deterministic point of view.

The most useful tool to that end is known as Lawler's loop erasing procedure. It consists in eliminating any cycle from a walk as soon as this cycle is closed while walking along the walk, e.g. walk $1 \to 2 \to 3 \to 3 \to 1 \to 3 \to 4$ would see cycles $3 \to 3$ (a self-loop) and $1 \to 2 \to 3 \to 1$ (a triangle) erased, thereby leaving the skeleton $1 \to 3 \to 4$. All cycles erased this way are simple, in particular on lattices they are SAP. As it turns out the set of erased cycles from a walk is sufficient to reconstruct that walk uniquely and the combinatorics of that construction is closely associated with Viennot's heaps of pieces theory (G. X. Viennot, *Heaps of pieces, I : Basic definitions and combinatorial lemmas*, Combinatoire énumérative. Lecture Notes in Mathematics, vol 1234). This observation allows for precise combinatorial counts such as enumerating walks by their last erased cycle. From there one accesses the fraction $F(p)$ of *all* walks on a lattice whose last erased loop is some chosen SAP p. The behavior of $F(p)$ as the length ℓ of the SAP p increases is closely related to the connective constant problem mentioned above.

Recent results (P.-L. Giscard, Discrete Mathematics 344(4): 112305 (2021), *Asymptotic counts for the walk multiples of self-avoiding polygons on lattices using sieves*, also arXiv:2001.02084) provide an exact explicit formula giving $F(p)$ for any p on lattices, in particular on the square lattice. The goal of the present postdoctoral project is to obtain asymptotic estimates for $F(p)$ from the exact formula as the length ℓ of p increases using tools from the study of Laplacian determinants, S. Finski *Spanning trees, cycle-rooted spanning forests on discretizations of flat surfaces and analytic torsion*, arXiv:2001.05162; and

R. L. Greenblatt *Discrete and zeta-regularized determinants of the Laplacian on polygonal domains with Dirichlet boundary conditions*, arXiv:2102.04837v4.

Context

The postdoctoral position is jointly-funded through the ALCOHOL French National Research Agency research project and the Université du Littoral Côte d'Opale. The ALCOHOL (ALgebraic COmbinatorics of Hikes On Lattices) project aims at studying advanced algebraic and combinatorial structures associated with walks and heaps of cycles on graph. ALCOHOL is conducted by the maître de conférences Pierre-Louis Giscard at the Université du Littoral Côte d'Opale, Calais, France.

The postdoctoral position fully funded for a duration of **1 year** from January 2024 until January 2025. The position will be taken at the Université du Littoral Côte d'Opale, Calais, France.

Profile

The candidate should hold a **PhD in mathematics**, especially in the fields of combinatorics. Some knowledge of the theory of heaps of pieces is a bonus.

Period: 01/2024–01/2025

Salary: c. 3 890€/month gross salary

Application deadline: 15/06/2023

Application documents: Curriculum Vitae, recommendation(s) letter(s), PhD diploma and short description of your research.

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